Commodity dependence and optimal asset allocation

Vianney Dequiedt[a] Mathieu Gomes[a] Kuntara Pukthuanthong[b]
Benjamin Williams-Rambaud[a]

[a]Université Clermont Auvergne, France

[b]Robert Trulaske Sr. College of Business, University of Missouri, USA

August 13, 2024

Do commodities bring diversification benefits to a portfolio of traditional assets?

- Seminal works of Bodie and Rosansky (1980) and Gorton and Rouwenhorst (2006).
- Most studies take the perspective of a US investor or an investor in US dollar-denominated assets.

Our contribution to the existing literature

- We develop a small-economy model comprising a commodity- and a non-commodity-sector through which we derive a series of implications that lead us to contend that the extent to which a country is exposed to commodity risk should impact the benefits, or lack thereof, of incorporating commodities into the portfolios of domestic investors.
- Based on a broad set of countries, we empirically show that a nation's degree of commodity dependence affects the diversification benefits of commodities in a portfolio of conventional assets.

The model

We consider a two-sector, small and open economy. Sector 1 is the commodity sector while Sector 2 is the non-commodity sector.

Profit equations:

$$\Pi_1 = p_1 f_1(K_1, L_1) - r_1 K_1 - wL_1$$

$$\Pi_2 = p_2 f_2(K_2, L_2, q_2) - r_2 K_2 - wL_2 - p_1 q_2$$

The total amount of capital available in this economy is $\bar{K}$ which is split between the two sectors, i.e., $K1 + K2 = \bar{K}$.

We will also consider that there is no unemployment and that the total available quantity of labor is $\bar{L}$, so that $L1 + L2 = \bar{L}$.

Long-run equilibrium conditions

$$w^* = p_1 \frac{\partial f_1}{\partial L}(K_1^*, L_1^*), \tag{1}$$

$$w^* = p_2 \frac{\partial f_2}{\partial L}(K_2^*, L_2^*, q_2^*). \tag{2}$$

$$p_1 = p_2 \frac{\partial f_2}{\partial q}(K_2^*, L_2^*, q_2^*). \tag{3}$$

$$r^* = p_1 \frac{\partial f_1}{\partial K}(K_1^*, L_1^*), \tag{4}$$

$$r^* = p_2 \frac{\partial f_2}{\partial K}(K_2^*, L_2^*, q_2^*). \tag{5}$$

$$\bar{K} = K_1^* + K_2^*, \tag{6}$$

$$\bar{L} = L_1^* + L_2^*. \tag{7}$$

$$p_2 \frac{\partial^2 f_2}{\partial q^2}(K_2^*, L_2^*, q_2^*) \left(p_1 \frac{\partial^2 f_1}{\partial L^2}(K_1^*, L_1^*) + p_2 \frac{\partial^2 f_2}{\partial L^2}(K_2^*, L_2^*, q_2^*) \right) - \left(p_2 \frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*) \right)^2 \geq 0. \tag{8}$$

Short-run responses to shocks

$$w^* + \tilde{w} = (p_1 + \tilde{p}_1)\frac{\partial f_1}{\partial L}(K_1^*, L_1^* + \tilde{L}), \tag{9}$$

$$w^* + \tilde{w} = (p_2 + \tilde{p}_2)\frac{\partial f_2}{\partial L}(K_2^*, L_2^* - \tilde{L}, q_2^* + \tilde{q}). \tag{10}$$

$$p_1 + \tilde{p}_1 = (p_2 + \tilde{p}_2)\frac{\partial f_2}{\partial q}(K_2^*, L_2^* - \tilde{L}, q_2^* + \tilde{q}). \tag{11}$$

$$r^* + \tilde{r}_1 = (p_1 + \tilde{p}_1)\frac{\partial f_1}{\partial K}(K_1^*, L_1^* + \tilde{L}), \tag{12}$$

$$r^* + \tilde{r}_2 = (p_2 + \tilde{p}_2)\frac{\partial f_2}{\partial K}(K_2^*, L_2^* - \tilde{L}, q_2^* + \tilde{q}). \tag{13}$$

Linear approximations of short-term responses

$$\tilde{w} = \tilde{p}_1 \frac{\partial f_1}{\partial L}(K_1^*, L_1^*) + \tilde{L} p_1 \frac{\partial^2 f_1}{\partial L^2}(K_1^*, L_1^*), \tag{14}$$

$$\tilde{w} = \tilde{p}_2 \frac{\partial f_2}{\partial L}(K_2^*, L_2^*, q_2^*) - \tilde{L} p_2 \frac{\partial^2 f_2}{\partial L^2}(K_2^*, L_2^*, q_2^*) + \tilde{q} p_2 \frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*), \tag{15}$$

$$\tilde{p}_1 = \tilde{p}_2 \frac{\partial f_2}{\partial q}(K_2^*, L_2^*, q_2^*) - \tilde{L} p_2 \frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*) + \tilde{q} p_2 \frac{\partial^2 f_2}{\partial q^2}(K_2^*, L_2^*, q_2^*), \tag{16}$$

$$\tilde{r}_1 = \tilde{p}_1 \frac{\partial f_1}{\partial K}(K_1^*, L_1^*) + \tilde{L} p_1 \frac{\partial^2 f_1}{\partial K \partial L}(K_1^*, L_1^*), \tag{17}$$

$$\tilde{r}_2 = \tilde{p}_2 \frac{\partial f_2}{\partial K}(K_2^*, L_2^*, q_2^*) - \tilde{L} p_2 \frac{\partial^2 f_2}{\partial K \partial L}(K_2^*, L_2^*, q_2^*) + \tilde{q} p_2 \frac{\partial^2 f_2}{\partial K \partial q}(K_2^*, L_2^*, q_2^*). \tag{18}$$

Linear approximations of short-term responses

Proposition 1. *The short-run movements of returns on capital in both sectors in response to shocks on international commodity and non-commodity prices can be approximated by the linear formulae*

$$\begin{cases} \tilde{r}_1 = \alpha_1^1 \tilde{p}_1 + \alpha_2^1 \tilde{p}_2, \\ \tilde{r}_2 = \alpha_1^2 \tilde{p}_1 + \alpha_2^2 \tilde{p}_2, \end{cases} \tag{19}$$

where $\alpha_1^1 \geq 0$, $\alpha_2^2 \geq 0$, $\alpha_2^1 \leq 0$ and $\alpha_1^2 \leq 0$.

Covariances

We are interested in the returns of a representative portfolio of assets

$$r_r(s) = sr_1 + (1-s)r_2,$$

The short-run response to shocks on p_1 and p_2 of this average return will be denoted by

$$\tilde{r}_r(s) \equiv s\tilde{r}_1 + (1-s)\tilde{r}_2,$$

Proposition 2. *When world prices p_1 and p_2 are subject to small independent shocks $\tilde{p}_1$ and $\tilde{p}_2$ with zero mean, small variance and negligible higher moments, the short-run price responses $\tilde{r}_1, \tilde{r}_2$ and $\tilde{r}_r(s)$ are such that:*

1. $Cov(\tilde{p}_i, \tilde{r}_i) \geq 0$, for $i = 1, 2$,
2. $Cov(\tilde{p}_i, \tilde{r}_j) \leq 0$, for $i \neq j$,
3. $Cov(\tilde{r}_i, \tilde{r}_j) \leq 0$, for $i \neq j$,
4. $Cov(p_1, \tilde{r}_r(s))$ is positive for s close to 1, is negative for s close to 0, and is increasing in s for $0 \leq s \leq 1$.

Sample selection

- MSCI equity market indexes.
- Ten-year yields on government bonds from Refinitv.
- Availability of at least ten years of uninterrupted data.
- Panel of 38 countries between 2000 and 2020.

Commodities data

- Dow Jones Commodity Total Return Index.
- Mirrors a fully collateralized investment in nearby commodity futures, assumming positions are rolled over five days.

Commodity dependence

- Trade data for exports from the United Nation Conference on Trade and Development (UNCTAD) statistics.
- Commodity dependence assessed by dividing the total value of commodity exports by the total value of merchandise exports.
- Sample mean commodity dependence (32.2%) is used as a cut-off.

Country classification

- Group 1 includes low-commodity dependence countries, i.e., countries for which commodity risk exposure is low to moderate (below 32.2%, 24 countries).
- Group 2 includes high-commodity dependence countries, i.e., countries for which commodity risk exposure is moderate to high (above 32.2%, 14 countries).

Theoretical framework

- Classic mean-variance optimization approach (Markowitz, 1952).
- A comparison of the optimal Sharpe ratios (Sharpe, 1966) is conducted in-sample to evaluate the shift in the optimal portfolio when adding commodities.
- Both short positions and leverage are not allowed.

Hypothesis testing

- We use of a simple test of mean comparison (Z-test) to assess the statistical significance of the improvement in Sharpe ratios.

Results

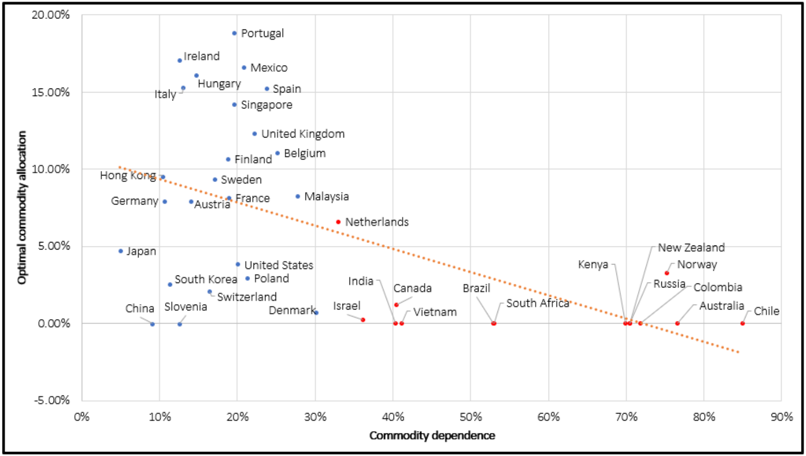
<u>The diversifying benefits of commodities depend on countries' degree of commodity dependenc</u>

- **Group 1:** in most low-commodity dependence countries (17 out of 24, or 71%), adding commodities to the asset mix improves the Sharpe ratio of the optimal portfolio.
- **Group 2:** in high-commodity dependence countries, adding commodities only improves the Sharpe ratio of the optimal portfolio in the case of the Netherlands (1 out of 14, or 7%)

<u>Commodities weight in optimal portfolios depends on countries' degree of commodity depende</u>

- **Group 1:** the average weight of commodities in optimal portfolios equals 9.20% ranging from 0% (for China, Poland, Slovenia, and Thailand) to 18.84% for Portugal.
- **Group 2:** the average weight of commodities in optimal portfolios equals 0.81% ranging from 0% (for 10 out 14 countries to 6.61% for the Netherlands)

Results


Optimal commodity allocation
Commodity dependence
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
Portugal
Ireland
Mexico
Hungary
Spain
Italy
Singapore
United Kingdom
Belgium
Finland
Hong Kong
Sweden
Germany
Austria
France
Malaysia
Netherlands
Japan
United States
South Korea
Poland
Switzerland
China
Slovenia
Denmark
Israel
India
Canada
Vietnam
Brazil
South Africa
Kenya
New Zealand
Norway
Russia
Colombia
Australia
Chile

Results

Additional tests

- Accounting for the DJCTRI individual commodity bias.
- Computing commodity dependence using investable commodities only.
- Accounting for the impact of exchange rates.
- Assessing Commodity Diversification Benefits with R^2 (Pukthuanthong and Roll, 2009).

Discussion

Investors in countries that depend heavily on commodities typically do not gain from incorporating commodities into their portfolio of domestic assets. In contrast, those in countries with low commodity dependence usually do.

- Because economic growth is a crucial driver of public debt sustainability, the price of exported commodities should impact the returns of bonds issued by commodity-dependent countries.
- Commodity prices affect stock returns through the earnings of commodity firms, which are typically over-represented in the major indexes of commodity-dependent countries.
- High-commodity-dependent countries tend to have less diversified economies.
- Financial markets in high-commodity-dependent countries might be more susceptible to volatility spillover from commodity markets.

Commodity dependence and optimal asset allocation

Vianney Dequiedt[a] Mathieu Gomes[a] Kuntara Pukthuanthong[b]
Benjamin Williams-Rambaud[a]

[a]Université Clermont Auvergne, France

[b]Robert Trulaske Sr. College of Business, University of Missouri, USA

August 13, 2024