Commodity Prices, Discount Rates, and Expected Dividend Growth

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motivation

- Wikipedia: commodity is an economic good, usually a resource, that specifically has full or substantial fungibility: that is, the market treats instances of the good as equivalent or nearly so with no regard to who produced them.
- commodities are both production inputs and investment assets that serve multiple investing purposes (hedging, speculation, arbitrage)
- commodity prices should reflect:
  - expected stock market returns (i.e., discount rates), μ_t
  - expected stock market cash-flow growth rates, g_t

  two forces at the center of leading asset pricing theories
- this paper studies how to extract the latter from commodity prices

main results

- I show that just gold and oil prices are enough to infer those states
- I show regression-based, PCA, and state-space model evidence that: (μ_t, g_t annualized in percentage):

$$\log G_t = const + \frac{1}{40}\mu_t + \frac{1}{4}g_t + \epsilon_t^G \quad (1)$$

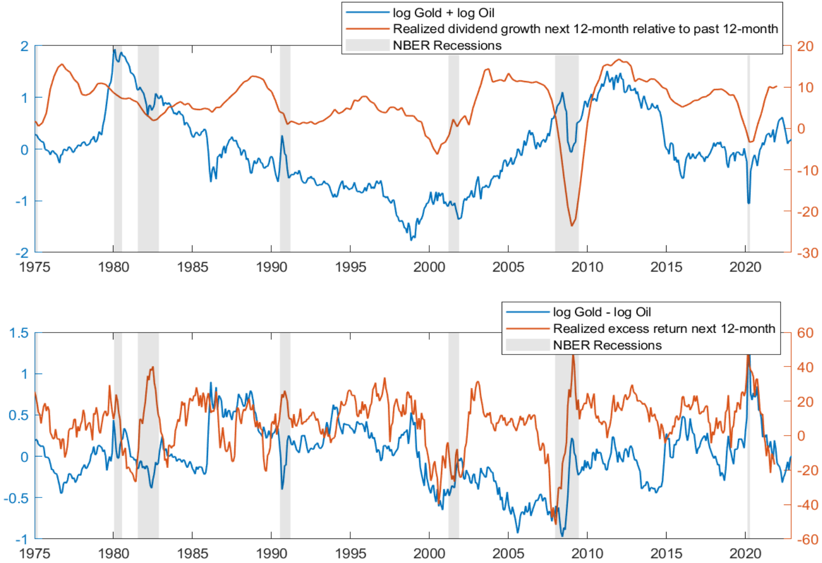
$$\log O_t = const - \frac{1}{40}\mu_t + \frac{1}{4}g_t + \epsilon_t^O \quad (2)$$

$$r_{t+1} = \mu_t + \varepsilon_{r,t+1} \quad (3)$$

$$\Delta d_{t+1} = g_t + \varepsilon_{d,t+1} \quad (4)$$

- $\log G_t - \log O_t$ is a return predictor (annual IS $R^2 = 11\%$ and OOS $R^2 = 14\%$), negatively priced in cross-section
- $\log G_t + \log O_t$ is a dividend growth rate predictor (annual IS $R^2 = 6\%$ and OOS $R^2 = 8\%$), positively priced in cross-section

main results



main results

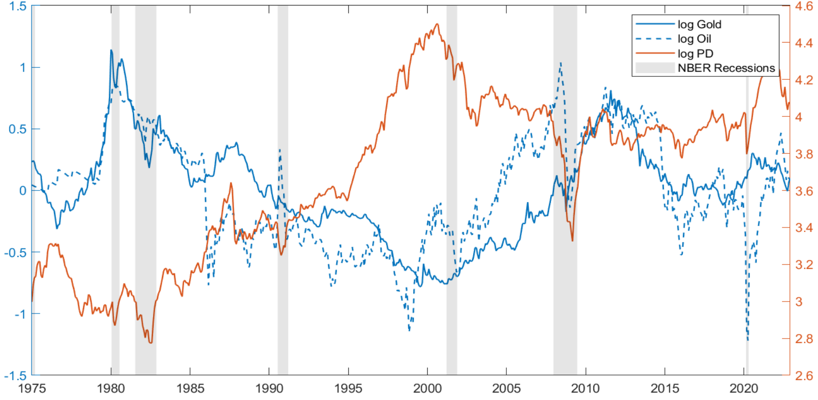
- the paper also sheds light on the "omitted variable bias (OVB)" or "error-in-variable bias" in time-series return predictive regressions
- suppose we want to study the relation between log gold price and expected stock returns, and we run future stock returns directly onto log gold price:

$$r_{t+1} = const + \beta_1 \log G_t + \underbrace{\tilde{\varepsilon}_{t+1}} \tag{5}$$

$$= const + \beta_1(\frac{1}{40}\mu_t + \frac{1}{4}g_t) \underbrace{-\beta_1 \frac{1}{4}g_t + \varepsilon_{r,t+1}} . \tag{6}$$

- the true relation is positive
- but our slope coefficient β_1 can be zero or negative due to the OVB
- there are two components of the OVB:
  - the "attenuation bias" biases β_1 toward zero
  - the covariance between μ_t and g_t is negative, which biases β_1 toward a negative territory
- I estimate OVB equal to 100%

main results



the OVB helps explain the literature findings

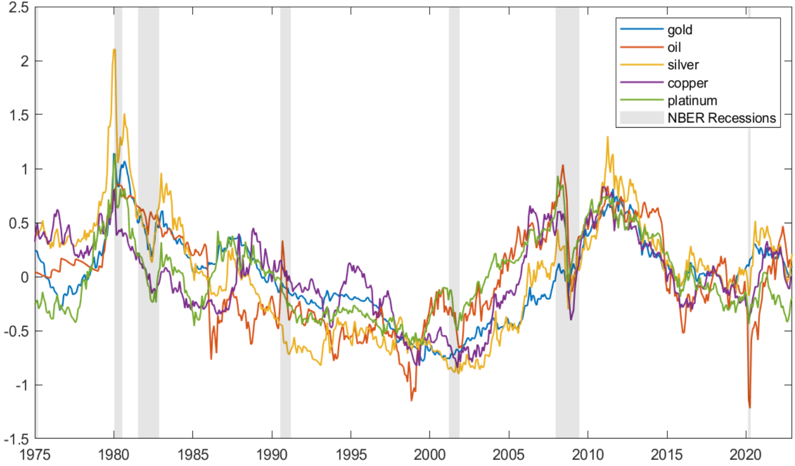
- the literature almost agrees that oil price negatively predicts stock returns (positively correlates with stock returns):
  - Chen, Roll, and Ross (1986), Jones and Kaul (1996), Driesprong, Jacobsen, and Maat (2008), Huang, Masulis, and Stoll (1996), Sim and Zhou (2015), Ready (2018), Christoffersen and Pan (2018), Gao, Hitzemann, Shaliastovich, and Xu (2022)

- the literature vastly disagrees on how gold price predicts stock returns (correlates with stock returns):
  - Barro and Misra (2016), Baur and Smales (2020): hedging asset
  - Huang and Kilic (2019), Hou, Tang, and Zhang (2020): risky asset
  - Erb and Harvey (2013): no relation
- I reconcile different opinions on gold

other omitted variables?

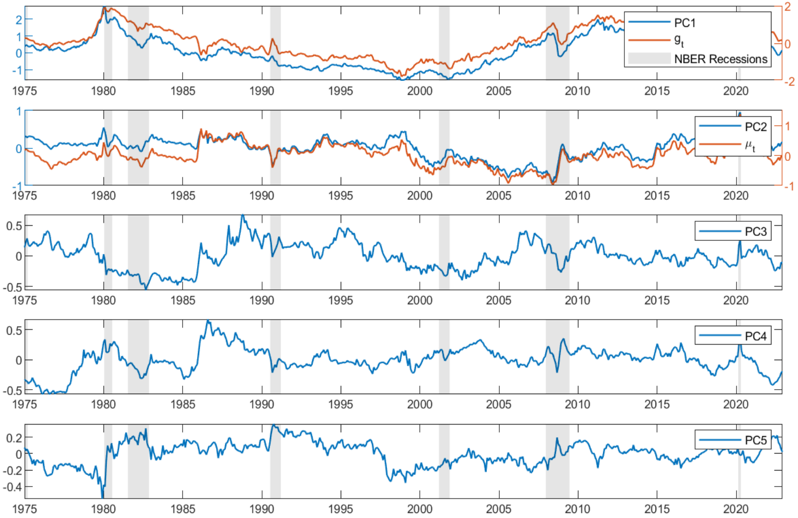
- commodity prices are driven by other forces not captured by μ_t, g_t like inflation, hedging demand, sentiments, extrapolation, etc
- model overfitting?
- unlikely
- through PCA, I show that μ_t and g_t almost span the state-space of commodity pricing

PCA



- only consider metal, energy and ignore livestock, agricultural because the latter behave a bit differently

principal components



- PC1 is basically g_t; PC2 is basically μ_t

commodities' loadings on PCs

- Commodities' loadings on PCs are given by

$$\begin{pmatrix} & PC(1) & PC(2) & PC(3) & PC(4) & PC(5) \\ \text{variation explained} & 79\% & 9\% & 5\% & 5\% & 2\% \\ Gold & 0.41 & 0.44 & -0.05 & 0.28 & 0.75 \\ Oil & 0.45 & -0.62 & -0.48 & -0.36 & 0.21 \\ Silver & 0.61 & 0.49 & -0.19 & -0.16 & -0.57 \\ Copper & 0.37 & -0.16 & 0.84 & -0.35 & 0.08 \\ Platinum & 0.35 & -0.40 & 0.13 & 0.80 & -0.24 \end{pmatrix} \tag{7}$$

the paper also contributes to return/dividend growth predictability literature

- Cochrane (2008), Cochrane (2011), Koijen and Van Nieuwerburgh (2011), and Van Binsbergen and Koijen (2010), Kelly and Pruitt (2013), Pruitt (2023), De La O and Myers (2021), Li and Wang (2024), etc
- using state-space model, Van Binsbergen and Koijen (2010) find that both returns and dividend growth rates are predictable by information contained in PD ratio
- I show that commodity prices contain information that predicts returns and dividend growth rates, above and beyond PD ratio

data

- Jan 1975 to Dec 2022
- monthly (and daily) commodity price data from Macrotrends
- monthly dividend and price-dividend ratio data from Amit Goyal's (Robert Shiller's) website
- monthly (and daily) stock market return data from Ken French's website

- commodity prices are non-stationary
- but log commodity prices cointegrate with log dividends
- detrend using log dividends

data

Table 1: Johansen (1988) Rank Test

We estimate an Engle and Granger (1987) vector error-correction model (VECM)

$$\Delta Y_t = \mu + \Pi Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t$$

and conduct Johansen (1988) rank test for cointegration based on the rank of the matrix Π. The null hypothesis for the rank test is that there are no more than r cointegration relations.

	statistic	c-value	p-value
$Y_t = [\log G_t, \log O_t]'$			
$H_0 : r = 0$	12.6	15.5	0.129
$H_0 : r = 1$	1.0	3.8	0.454
$Y_t = [\log G_t, \log D_t]'$			
$H_0 : r = 0$	16.9	15.5	0.031
$H_0 : r = 1$	2.6	3.8	0.111
$Y_t = [\log O_t, \log D_t]'$			
$H_0 : r = 0$	23.9	15.5	0.003
$H_0 : r = 1$	5.1	3.8	0.024
$Y_t = [\log G_t/O_t, \log D_t]'$			
$H_0 : r = 0$	28.8	15.5	0.001
$H_0 : r = 1$	8.1	3.8	0.005
$Y_t = [\log G_t \cdot O_t, \log D_t]'$			
$H_0 : r = 0$	19.2	15.5	0.013
$H_0 : r = 1$	3.0	3.8	0.083
$Y_t = [\log G_t, \log O_t, \log D_t]'$			
$H_0 : r = 0$	33.4	29.8	0.018
$H_0 : r = 1$	10.5	15.5	0.269
$H_0 : r = 2$	2.3	3.8	0.129

data

Table 2: Stock and Watson (1993) dynamic least square

$$\log X_t = \beta_0 + \beta_D \log D_t + \sum_{i=-k}^{i=k} \gamma_i \Delta \log D_{t-i} + \epsilon_t$$

	estimate	s.e.	t-stat	[95% CI]
		$X = G$		
$\beta_D(k=1)$	0.62	0.06	10.54	[0.50, 0.73]
$\beta_D(k=2)$	0.62	0.07	8.58	[0.48, 0.76]
$\beta_D(k=3)$	0.62	0.08	7.46	[0.46, 0.78]
		$X = O$		
$\beta_D(k=1)$	0.16	0.08	1.96	[0.00, 0.32]
$\beta_D(k=2)$	0.16	0.09	1.62	[-0.03, 0.35]
$\beta_D(k=3)$	0.16	0.11	1.44	[-0.06, 0.38]
		$X = G/O$		
$\beta_D(k=1)$	0.46	0.08	5.64	[0.30, 0.62]
$\beta_D(k=2)$	0.46	0.10	4.76	[0.27, 0.65]
$\beta_D(k=3)$	0.46	0.11	4.25	[0.25, 0.67]
		$X = G \cdot O$		
$\beta_D(k=1)$	0.78	0.12	6.66	[0.55, 1.01]
$\beta_D(k=2)$	0.78	0.14	5.47	[0.50, 1.05]
$\beta_D(k=3)$	0.78	0.16	4.80	[0.46, 1.10]

data

- estimated cointegration relation:

$$\begin{aligned} \log G_t &\equiv \log G_t - 0.62 \log D_t \\ \log O_t &\equiv \log O_t - 0.16 \log D_t \\ \log G_t/O_t &\equiv \log G_t - \log O_t - 0.46 \log D_t \\ \log G_t \cdot O_t &\equiv \log G_t + \log O_t - 0.78 \log D_t. \end{aligned}$$

data

Table 3: Summary statistics

Variable	Std	AR(1)	ADF p.val	Start	End
$\log G \equiv \log G_t - 0.62 \log D_t$	0.39	0.992	0.059	1975.1	2022.12
$\log O \equiv \log O_t - 0.16 \log D_t$	0.45	0.976	0.005	1975.1	2022.12
$\log G/O \equiv \log G_t/O_t - 0.46 \log D_t$	0.36	0.957	0.000	1975.1	2022.12
$\log G \cdot O \equiv \log G_t \cdot O_t - 0.78 \log D_t$	0.76	0.989	0.036	1975.1	2022.12
$\log\ G_t/P_t$	0.35	0.978	0.023	1975.1	2022.12
VRP_t	22.42	0.329	0.000	1990.1	2019.12
$\log\ PD_t$	0.44	0.994	0.073	1975.1	2022.12
$Sentiment_t^{BW}$	0.87	0.981	0.010	1975.1	2022.6
$Sentiment_t^{PLS}$	0.96	0.988	0.035	1975.1	2020.12
$Risk\_Aversion_t^{BEX}$	0.67	0.788	0.000	1986.7	2022.12
VIX_t	7.56	0.808	0.000	1990.1	2022.12
$Interest\_Rate_t$	0.29	0.978	0.005	1975.1	2022.12
$Inflation_t$	0.14	0.644	0.000	1975.1	2022.12
$DFSP_t$	0.45	0.959	0.000	1975.1	2022.12
$TMSP_t$	1.12	0.952	0.001	1975.1	2022.12
ICC_t	3.10	0.983	0.035	1977.1	2017.12
$Skew_t^Q$	0.41	0.847	0.000	1996.1	2021.12
$Kurt_t^Q$	1.11	0.863	0.000	1996.1	2021.12
$Crash\_Prob_t$	2.70	0.776	0.000	1996.1	2021.12

return predictive regressions

▸ model 1

▸ model 2

Table 4: Univariate Return Predictability: Gold and Oil

$$\frac{12}{h}\sum_{i=1}^{h}(r_{t+i}-r^f_{t+i}) = \beta_0 + \beta_X X_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G/O$				
$\beta_{\log G/O}$	20.03	18.17	18.02	16.73	15.54	14.48	8.01	5.32
s.e.	[6.67]	[6.53]	[6.65]	[6.91]	[6.41]	[5.88]	[3.77]	[2.53]
R^2_{adj} (%)	1.58	2.63	4.08	7.31	9.46	11.04	11.74	9.78
				$X = \log G \cdot O$				
$\beta_{\log G \cdot O}$	-1.25	-0.86	-0.88	-0.91	-0.95	-0.71	1.88	2.29
s.e.	[3.17]	[3.04]	[3.00]	[3.03]	[2.98]	[2.83]	[2.10]	[1.49]
R^2_{adj} (%)	-0.14	-0.15	-0.13	-0.08	-0.01	-0.06	3.11	9.10
				$X = \log G$				
$\beta_{\log G}$	6.15	6.10	5.99	5.39	4.78	4.80	6.63	6.37
s.e.	[6.06]	[5.69]	[5.51]	[5.43]	[5.51]	[5.57]	[4.05]	[2.73]
R^2_{adj} (%)	0.02	0.20	0.38	0.74	0.90	1.27	10.57	18.72
				$X = \log O$				
$\beta_{\log O}$	-8.18	-7.03	-7.01	-6.64	-6.34	-5.66	0.42	1.83
s.e.	[5.42]	[5.31]	[5.35]	[5.54]	[5.28]	[4.76]	[3.62]	[2.69]
R^2_{adj} (%)	0.28	0.48	0.84	1.67	2.33	2.50	-0.13	1.81

return predictive regressions

Table 5: Bivariate Return Predictability: I

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^f_{t+i}\right)=\beta_0+\beta_{\log G/O}\log G_t/O_t+\beta_X X_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G/O}$	17.22	14.84	14.52	12.31	10.36	8.59	2.58	0.20
s.e.	[8.22]	[7.90]	[7.87]	[7.39]	[6.26]	[5.22]	[2.36]	[1.83]
$\beta_{\log G/P}$	5.70	6.77	7.09	8.87	10.18	11.48	12.54	11.93
s.e.	[7.73]	[7.34]	[7.25]	[7.25]	[7.13]	[6.26]	[3.00]	[1.74]
R^2_{adj} (%)	1.51	2.74	4.38	8.57	11.94	15.19	28.31	31.74
$\beta_{\log G/O}$	18.08	16.09	16.03	14.61	13.24	12.15	7.20	4.48
s.e.	[7.17]	[7.09]	[7.19]	[7.29]	[6.56]	[5.78]	[3.53]	[2.15]
$\beta_{\log PD}$	-13.84	-14.85	-14.42	-15.29	-15.98	-16.32	-13.87	-14.12
s.e.	[9.90]	[9.41]	[9.02]	[8.02]	[7.39]	[6.73]	[3.91]	[1.59]
R^2_{adj} (%)	1.74	3.43	5.43	10.88	15.39	19.56	39.57	57.63
$\beta_{\log G/O}$	20.54	18.65	18.46	17.14	15.92	14.85	8.93	5.89
s.e.	[6.53]	[6.39]	[6.53]	[6.85]	[6.32]	[5.75]	[3.75]	[2.60]
$\beta_{I.R.}$	-9.42	-8.97	-8.17	-7.71	-7.67	-7.61	-8.10	-4.94
s.e.	[7.27]	[7.12]	[7.24]	[7.26]	[6.96]	[6.06]	[3.74]	[4.05]
R^2_{adj} (%)	1.66	2.91	4.49	8.19	10.83	12.89	19.88	15.36
$\beta_{\log G/O}$	19.38	17.82	17.69	16.27	14.94	13.94	7.86	5.23
s.e.	[6.82]	[6.75]	[6.85]	[6.90]	[6.32]	[5.79]	[3.79]	[2.54]
$\beta_{Inflation}$	-20.72	-10.76	-10.58	-14.43	-20.44	-19.20	-8.57	-5.41
s.e.	[17.60]	[17.28]	[15.87]	[9.30]	[9.68]	[8.91]	[3.74]	[4.07]
R^2_{adj} (%)	1.69	2.61	4.14	7.99	11.77	13.77	13.72	11.21

return predictive regressions

Table 6: Bivariate Return Predictability: II

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_{\log G/O}\log G_t/O_t+\beta_X X_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G/O}$	19.94	18.09	17.94	16.60	15.41	14.37	7.96	5.24
s.e.	[6.77]	[6.64]	[6.79]	[7.08]	[6.53]	[5.96]	[3.77]	[2.60]
β_{DFSP}	3.42	2.85	3.41	5.12	4.99	4.56	1.88	3.35
s.e.	[7.32]	[6.79]	[6.21]	[4.87]	[4.23]	[3.65]	[2.38]	[1.96]
R^2_{adj} (%)	1.49	2.57	4.15	8.25	10.87	12.63	12.72	16.47
$\beta_{\log G/O}$	19.63	17.88	17.61	16.08	15.10	14.33	7.80	4.78
s.e.	[6.92]	[6.80]	[7.00]	[7.35]	[6.86]	[6.24]	[3.34]	[2.40]
β_{TMSP}	0.28	0.27	0.35	0.43	1.19	1.67	2.88	2.40
s.e.	[2.33]	[2.13]	[2.06]	[1.79]	[1.62]	[1.48]	[1.09]	[0.85]
R^2_{adj} (%)	1.48	2.63	4.10	7.13	9.96	12.91	27.56	27.31
$\beta_{\log G/O}$	17.62	17.90	18.28	20.48	20.30	20.42	14.44	9.00
s.e.	[7.19]	[7.57]	[8.13]	[9.51]	[8.77]	[7.78]	[2.79]	[2.28]
β_{VRP}	0.51	0.39	0.35	0.15	0.05	0.02	-0.04	-0.05
s.e.	[0.09]	[0.08]	[0.06]	[0.05]	[0.05]	[0.05]	[0.02]	[0.02]
R^2_{adj} (%)	7.26	9.63	12.96	14.11	16.58	20.88	28.79	21.00

return predictive regressions

Table 7: Bivariate Return Predictability: III

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_{\log G/O}\log G_t/O_t+\beta_X X_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G/O}$	25.77	24.29	24.29	23.05	21.95	21.15	14.02	9.10
s.e.	[7.85]	[7.80]	[8.17]	[8.59]	[7.67]	[6.84]	[2.91]	[2.22]
β_{VIX}	0.37	0.50	0.46	0.44	0.32	0.27	0.06	0.02
s.e.	[0.55]	[0.45]	[0.42]	[0.24]	[0.22]	[0.20]	[0.22]	[0.22]
R^2_{adj} (%)	3.03	6.25	9.61	17.16	21.67	25.83	26.98	21.02
$\beta_{\log G/O}$	32.26	31.96	33.14	31.24	28.72	26.87	14.13	5.59
s.e.	[7.66]	[6.85]	[7.16]	[8.30]	[7.61]	[6.69]	[4.21]	[4.28]
β_{Skew^Q}	2.42	2.90	4.34	3.95	3.51	3.59	-1.67	-3.82
s.e.	[8.49]	[7.65]	[7.24]	[5.76]	[5.52]	[5.29]	[3.62]	[3.18]
R^2_{adj} (%)	3.98	7.89	13.17	22.48	27.24	30.42	24.82	14.91
$\beta_{\log G/O}$	32.66	32.82	33.30	31.01	28.15	26.34	14.03	5.84
s.e.	[7.84]	[7.16]	[7.38]	[8.29]	[7.41]	[6.53]	[3.98]	[4.20]
β_{Kurt^Q}	-1.93	-2.91	-2.58	-1.73	-0.98	-1.07	0.91	1.41
s.e.	[2.54]	[2.38]	[2.21]	[1.58]	[1.51]	[1.46]	[1.32]	[1.40]
R^2_{adj} (%)	4.10	8.46	13.67	22.70	27.04	30.21	25.36	14.45
$\beta_{\log G/O}$	32.82	30.56	31.38	30.39	27.81	25.98	14.56	6.84
s.e.	[8.65]	[8.32]	[8.51]	[8.58]	[7.66]	[6.90]	[3.25]	[3.76]
$\beta_{Crash-Prob}$	-0.85	0.28	0.19	-0.29	-0.16	-0.19	0.45	0.16
s.e.	[1.35]	[1.09]	[0.97]	[0.69]	[0.65]	[0.65]	[0.39]	[0.45]
R^2_{adj} (%)	4.12	7.85	12.92	22.15	26.79	29.84	25.62	10.93

return predictive regressions

Table 8: Bivariate Return Predictability: IV

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_{\log G/O}\log G_t/O_t+\beta_X X_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G/O}$	20.03	18.13	18.18	16.93	15.78	14.73	8.01	5.32
s.e.	[6.54]	[6.40]	[6.50]	[6.72]	[6.16]	[5.59]	[3.76]	[2.53]
$\beta_{Sentiment^{BW}}$	-5.03	-4.70	-4.67	-4.38	-4.25	-3.72	-0.07	0.11
s.e.	[2.26]	[2.09]	[1.99]	[1.88]	[1.90]	[1.99]	[1.14]	[0.94]
R^2_{adj} (%)	2.06	3.54	5.59	10.19	13.53	15.11	11.59	9.64
$\beta_{\log G/O}$	14.92	13.40	13.62	13.62	13.12	12.64	7.49	5.59
s.e.	[6.67]	[6.59]	[6.74]	[7.19]	[6.75]	[6.20]	[4.02]	[2.37]
$\beta_{Sentiment^{PLS}}$	-6.46	-6.14	-5.60	-4.40	-3.73	-3.26	-0.68	0.37
s.e.	[1.93]	[1.74]	[1.66]	[1.84]	[2.02]	[2.07]	[1.29]	[0.72]
R^2_{adj} (%)	2.60	4.49	6.47	10.78	13.55	15.60	12.22	9.96
$\beta_{\log G/O}$	19.09	18.13	18.51	18.05	16.96	15.73	9.31	6.15
s.e.	[7.51]	[7.43]	[7.60]	[7.81]	[7.13]	[6.51]	[3.99]	[2.64]
$\beta_{RA^{BEX}}$	4.89	6.61	5.27	5.73	4.85	4.40	1.53	1.72
s.e.	[7.34]	[5.99]	[5.42]	[2.56]	[1.92]	[1.60]	[1.63]	[1.42]
R^2_{adj} (%)	1.85	4.12	6.34	13.37	16.88	19.27	16.87	15.84
$\beta_{\log G/O}$	14.97	13.10	13.12	13.15	12.13	11.34	7.27	4.38
s.e.	[7.20]	[7.30]	[7.53]	[7.93]	[7.07]	[6.28]	[3.57]	[2.51]
β_{ICC}	-0.00	0.16	0.13	0.20	0.24	0.27	0.41	0.43
s.e.	[0.86]	[0.87]	[0.88]	[0.87]	[0.78]	[0.65]	[0.39]	[0.34]
R^2_{adj} (%)	0.63	1.17	1.96	4.50	6.02	7.38	14.67	14.01

return predictive regressions

Table 9: Daily regressions

$$\frac{260}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_X X_t+\epsilon_{t+h}$$

	1d	2d	3d	4d	1w	2w
			$X = \log G/O$			
$\beta_{\log G/O}$	18.26	18.77	19.33	19.95	20.38	21.01
s.e.	[8.30]	[7.79]	[7.47]	[7.28]	[7.15]	[7.09]
R^2_{adj} (%)	0.05	0.12	0.19	0.28	0.38	0.69
			$X = Sentiment^{BW}$			
β_{BW}	-10.74	-10.71	-10.65	-10.63	-10.64	-10.65
s.e.	[5.26]	[4.98]	[4.82]	[4.71]	[4.63]	[4.56]
R^2_{adj} (%)	0.04	0.10	0.16	0.23	0.29	0.50

$$\frac{260}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_{\log G/O}\log G_t/O_t+\beta_{BW}Sentiment^{BW}_t+\epsilon_{t+h}$$

	1d	2d	3d	4d	1w	2w
$\beta_{\log G/O}$	17.95	18.44	18.97	19.58	20.04	20.78
s.e.	[8.28]	[7.77]	[7.44]	[7.25]	[7.11]	[7.04]
β_{BW}	-10.54	-10.51	-10.45	-10.42	-10.42	-10.43
s.e.	[5.23]	[4.95]	[4.79]	[4.68]	[4.59]	[4.51]
R^2_{adj} (%)	0.09	0.22	0.36	0.51	0.67	1.19

return predictive regressions

Table 10: A Three-Factor Model

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_{\log G/O}\log G_t/O_t+\beta_{Sentiment^{BW}}Sentiment^{BW}_t+\beta_{Skew^Q}Skew^Q_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G/O}$	31.62	31.33	32.51	30.45	27.83	25.97	13.49	5.07
s.e.	[7.65]	[6.83]	[7.12]	[8.15]	[7.34]	[6.22]	[4.17]	[4.00]
$\beta_{Sentiment^{BW}}$	-8.40	-8.39	-8.38	-10.43	-11.61	-11.80	-3.61	-3.08
s.e.	[3.94]	[3.32]	[2.85]	[2.27]	[1.96]	[1.60]	[1.51]	[2.59]
β_{Skew^Q}	4.85	5.33	6.77	6.97	6.88	7.01	0.05	-2.26
s.e.	[8.68]	[7.63]	[6.92]	[4.49]	[3.12]	[2.31]	[3.23]	[3.12]
R^2_{adj} (%)	4.76	9.66	16.04	31.38	43.16	51.53	29.81	22.66

uncertainty or risk-aversion?

Table 11: G/O and Other Variables

$$X_t = \beta_0 + \beta_{\log G/O} \log G_t/O_t + \epsilon_t$$

$X =$	$Sentiment^{PLS}$	ICC	$Skew^{Q}$	$Kurt^{Q}$	$\log G/P$	VRP
$\beta_{\log G/O}$	-0.67	1.99	-0.33	0.62	0.49	8.76
s.e.	[0.35]	[0.66]	[0.12]	[0.33]	[0.11]	[4.79]
R^2_{adj} (%)	6.12	5.08	8.67	3.96	24.90	1.40

results

- suppose we have an equilibrium model that prices gold and oil. Under reasonable cointegration assumptions and log-linearization, the model will imply the following two pricing equations:

$$\log G_t - c^G \log D_t = const + \beta_\mu^G \mu_t + \beta_g^G g_t + \epsilon_t^G \quad (8)$$

$$\log O_t - c^O \log D_t = const + \beta_\mu^O \mu_t + \beta_g^O g_t + \epsilon_t^O, \quad (9)$$

  where μ_t is expected returns and g_t is expected dividend growth rate
- one can think about long-run risk models
- our goal is to determine the values of coefficients β_μ^i, β_g^i
- I will show regression-based evidence that largely supports the following parameterization (μ_t and g_t are annualized in percentage):

$$\log G_t = const + \frac{1}{40}\mu_t + \frac{1}{4}g_t + \epsilon_t^G \quad (10)$$

$$\log O_t = const - \frac{1}{40}\mu_t + \frac{1}{4}g_t + \epsilon_t^O, \quad (11)$$

first test

- take the difference of the two equations to get:

$$\mu_t = const + 20 \log G_t/O_t + \epsilon_t, \tag{12}$$

- if we run future returns (over a short period) onto $\log G_t/O_t$, slope should be 20. Confirmed in Table 4
- if we run future returns onto $\log G_t$ and $\log O_t$ simultaneously, slopes on the two should be respectively 20 and -20. Confirmed in Table 12
- if we run future dividend growth rates onto $\log G_t/O_t$, slope should be insignificant. Confirmed in Table 13

second test

- take the sum of the two equations to get:

$$g_t = const + 2 \log G_t \cdot O_t + \epsilon_t, \tag{13}$$

  - if we run future dividend growth rates (over a short period) onto $\log G_t \cdot O_t$, slope should be 2. Confirmed in Table 13
  - if we run future dividend growth rates onto $\log G_t$ and $\log O_t$ simultaneously, slopes on the two should be respectively 2 and 2. Confirmed in Table 14
  - if we run future returns onto $\log G_t \cdot O_t$, slope should be insignificant. Confirmed in Table 4

third test

- we can also derive:

$$\mu_t = const + 40 \log G_t - 20 \log G_t \cdot O_t + \epsilon_t \tag{14}$$

$$\mu_t = const - 40 \log O_t + 20 \log G_t \cdot O_t + \epsilon_t, \tag{15}$$

- if we run future returns (over a short period) onto $\log G_t$ or $\log O_t$, but controlling for expected dividend growth as proxied by $\log G_t \cdot O_t$, then slopes should be 40 and -40

- Table 15 confirms this is the case

results

Table 12: Bivariate Return Predictability: Gold and Oil

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r^{f}_{t+i}\right)=\beta_0+\beta_{\log G}\log G_t+\beta_{\log O}\log O_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G}$	20.77	19.23	19.03	17.54	16.09	15.24	11.53	9.12
s.e.	[7.97]	[7.57]	[7.51]	[7.56]	[7.30]	[7.20]	[5.08]	[3.19]
$\beta_{\log O}$	-19.71	-17.70	-17.58	-16.38	-15.29	-14.14	-6.42	-3.60
s.e.	[6.91]	[6.83]	[6.97]	[7.29]	[6.70]	[5.97]	[3.83]	[2.78]
R^2_{adj} (%)	1.41	2.48	3.94	7.19	9.32	10.95	17.51	22.79

results

model 1
model 2

Table 13: Univariate Dividend Growth Predictability: Gold and Oil

$$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+i} = \beta_0 + \beta_X X_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G/O$				
$\beta_{\log G/O}$	-1.91	-1.83	-1.77	-1.38	-0.85	-0.17	2.24	0.44
s.e.	[1.39]	[1.52]	[1.63]	[1.96]	[2.26]	[2.54]	[2.54]	[1.81]
R^2_{adj} (%)	0.77	0.73	0.69	0.39	0.05	-0.17	0.28	0.05
				$X = \log G \cdot O$				
$\beta_{\log G \cdot O}$	2.10	2.11	2.13	2.13	2.10	2.01	1.45	1.36
s.e.	[0.67]	[0.70]	[0.73]	[0.80]	[0.88]	[0.96]	[1.17]	[0.90]
R^2_{adj} (%)	4.93	5.20	5.43	5.87	6.01	5.89	6.07	11.57
				$X = \log G$				
$\beta_{\log G}$	3.20	3.26	3.31	3.48	3.63	3.74	3.62	2.76
s.e.	[1.37]	[1.45]	[1.52]	[1.68]	[1.81]	[1.91]	[2.39]	[1.90]
R^2_{adj} (%)	2.92	3.16	3.37	4.05	4.70	5.37	10.03	12.51
				$X = \log O$				
$\beta_{\log O}$	3.62	3.61	3.61	3.49	3.27	2.93	1.49	1.89
s.e.	[1.10]	[1.16]	[1.22]	[1.38]	[1.59]	[1.82]	[2.23]	[1.55]
R^2_{adj} (%)	5.10	5.30	5.46	5.48	5.09	4.34	2.03	7.45

results

Table 14: Univariate Dividend Growth Predictability: Gold and Oil

$$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+i} = \beta_0 + \beta_{\log G}\log G_t + \beta_{\log O}\log O_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
$\beta_{\log G}$	0.88	0.99	1.07	1.52	2.05	2.68	4.54	2.41
s.e.	[1.75]	[1.89]	[2.03]	[2.37]	[2.65]	[2.90]	[3.33]	[2.30]
$\beta_{\log O}$	3.13	3.06	3.01	2.64	2.13	1.44	-1.21	0.45
s.e.	[1.40]	[1.51]	[1.62]	[1.95]	[2.29]	[2.63]	[2.70]	[1.82]
R^2_{adj} (%)	5.08	5.31	5.51	5.79	5.85	5.84	10.66	12.58

results

Table 15: Return Predictability: Controlling For Cash Flow Components

$$\frac{12}{h}\sum_{i=1}^{h}(r^M_{t+i} - r^f_{t+i}) = \beta_0 + \beta_X X_t + \beta_{\log G\cdot O}(\log G_t \cdot O_t) + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G$				
$\beta_{\log G}$	40.48	36.93	36.61	33.92	31.39	29.38	17.94	12.71
s.e.	[13.61]	[13.23]	[13.39]	[13.82]	[12.93]	[12.14]	[8.03]	[5.26]
$\beta_{\log G\cdot O}$	-19.71	-17.70	-17.58	-16.38	-15.29	-14.14	-6.42	-3.60
s.e.	[6.91]	[6.83]	[6.97]	[7.29]	[6.70]	[5.97]	[3.83]	[2.78]
R^2_{adj} (%)	1.41	2.48	3.94	7.19	9.32	10.95	17.51	22.79
				$X = \log O$				
$\beta_{\log O}$	-40.48	-36.93	-36.61	-33.92	-31.39	-29.38	-17.94	-12.71
s.e.	[13.61]	[13.23]	[13.40]	[13.82]	[12.93]	[12.14]	[8.03]	[5.26]
$\beta_{\log G\cdot O}$	20.77	19.23	19.03	17.54	16.09	15.24	11.53	9.12
s.e.	[7.97]	[7.57]	[7.51]	[7.56]	[7.30]	[7.20]	[5.08]	[3.19]
R^2_{adj} (%)	1.41	2.48	3.94	7.19	9.32	10.95	17.51	22.79

OOS evidence

- key insights so far: $\log G_t/O_t$ is a discount rate proxy, $\log G_t \cdot O_t$ is an expected cash flow growth proxy
- if this is the case, then
  - they should predict returns and dividend growth rates out-of-sample
  - they should be respectively negatively and positively priced in the cross-section of stock returns

OOS evidence

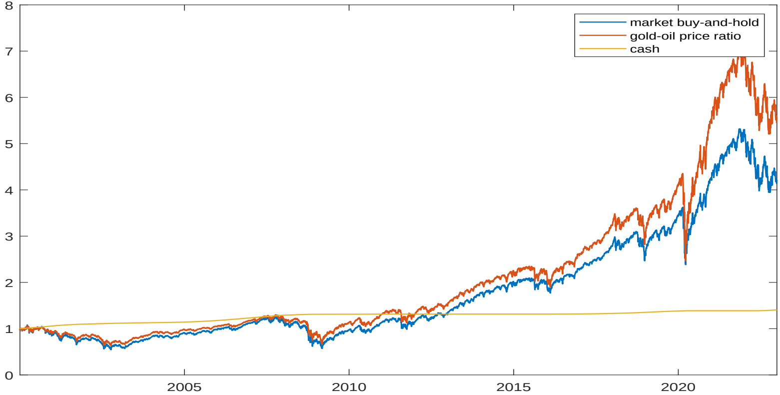
- split sample into 1975-1999, 2000-2022
- follow Welch and Goyal (2008) to compute OOS R-squared for return predictability by log G/O as

$$R^2_{OOS,R} = 1 - \frac{\sum_{t=0}^{T-1}(r_{t+1} - \hat{\mu}_t)^2}{\sum_{t=0}^{T-1}(r_{t+1} - \bar{r}_t)^2}, \tag{16}$$

  where $\hat{\mu}_t$ is the filtered value of the expected return using data only up until time t to estimate OLS model parameters. The denominator $\bar{r}_t$ is the historical mean of returns up until time t
- similar formula for dividend growth predictability by log GO

- OOS R-squared=0.03% for daily return, 3% for monthly return, 14% for annual return, 8% for annual dividend growth

OOS evidence



daily cumulative portfolio values for three strategies: stock market buy-and-hold, a strategy which sets the weight in market index proportional to the gold-oil price ratio (scaled to maintain the same standard deviation as market buy-and-hold), and a risk-free account.

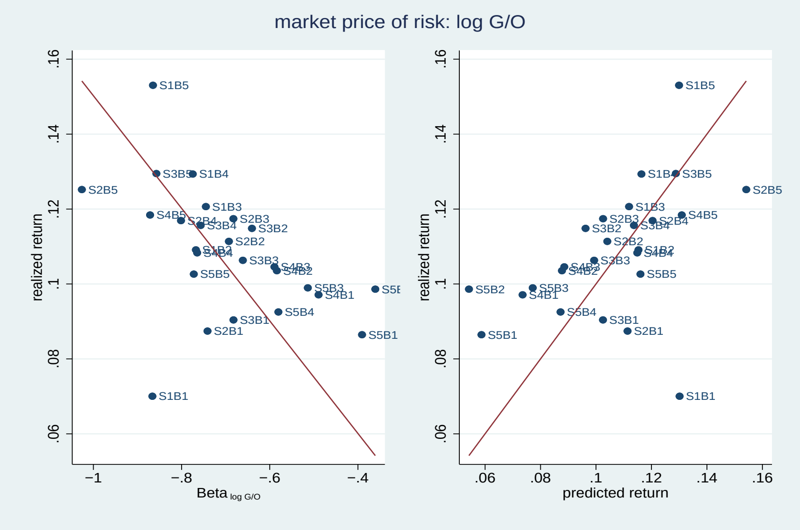
cross-sectional evidence

Table 16: Fama-Macbeth and Panel Regressions

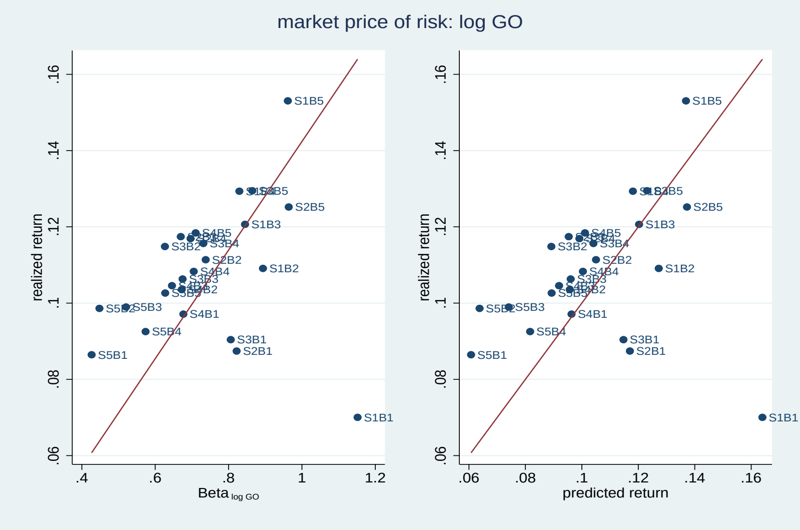
Test assets are Fama-French 100 size and book-to-market portfolios. A rolling window of 180 months is used to update portfolio's exposure to monthly AR(1) innovation in $\log G/O$ and $\log G \cdot O$.

Panel A: Fama-Macbeth Regressions			
$\lambda_{\Delta \log G/O}$	-0.14		-0.12
t-stat	[-3.48]		[-2.90]
$\lambda_{\Delta \log G \cdot O}$		0.10	0.07
t-stat		[2.88]	[2.28]
average R^2 (%)	8.23	6.74	12.56
Panel B: Panel Regressions			
$\lambda_{\Delta \log G/O}$	-0.09		-0.14
t-stat	[-4.91]		[-4.33]
$\lambda_{\Delta \log G \cdot O}$		0.16	0.14
t-stat		[8.83]	[7.21]
R^2 (%)	48.78	49.42	49.72

cross-sectional evidence



cross-sectional evidence



a state-space model

- so far, we've shown evidence based on short-horizon predictive regressions
  - we haven't taken long-horizon predictability into account
  - we haven't been able to identify the dynamics of μ_t and g_t
- to those ends and to quantify the OVB, we proceed to estimate a state-space model
- following Van Binsbergen and Koijen (2010), we assume μ_t and g_t respectively follow an AR(1)

a state-space model

- consider the model (which we estimate via MLE. we use Kalman filter to construct likelihood):

$$
\begin{aligned}
\log G_t &= \alpha^G + \beta_\mu^G(\mu_t - \bar{\mu}) + \beta_g^G(g_t - \bar{g}) + \varepsilon_{G,t} \\
\log O_t &= \alpha^O + \beta_\mu^O(\mu_t - \bar{\mu}) + \beta_g^O(g_t - \bar{g}) + \varepsilon_{O,t} \\
r_t &= \mu_{t-1} + \varepsilon_{r,t} \\
\Delta d_t &= g_{t-1} + \varepsilon_{d,t} \\
\mu_t &= (1 - \rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t} \\
g_t &= (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \varepsilon_{g,t}
\end{aligned}
$$

with

$$
Cov\left(\begin{bmatrix} \varepsilon_{G,t} \\ \varepsilon_{O,t} \\ \varepsilon_{r,t} \\ \varepsilon_{d,t} \end{bmatrix}\right) = \begin{bmatrix} \sigma_{G/O}^2 & & & \\ & \sigma_O^2 & & \\ & & \sigma_r^2 & \\ & & & \sigma_d^2 \end{bmatrix}
$$

$$
Cov\left(\begin{bmatrix} \varepsilon_{\mu,t} \\ \varepsilon_{g,t} \end{bmatrix}\right) = \begin{bmatrix} \sigma_\mu^2 & \sigma_{\mu g} \\ \sigma_{\mu g} & \sigma_g^2 \end{bmatrix}
$$

estimation results

- parameter estimation (s.e. in parathesis):

$$\log G_t = const + \underset{(0.016)}{0.036}(\mu_t - \underset{(4.2)}{12.5}) + \underset{(0.068)}{0.228}(g_t - \underset{(1.5)}{6.3}) + \varepsilon_{G,t}$$

$$\log O_t = const - \underset{(0.015)}{0.022}(\mu_t - \underset{(4.2)}{12.5}) + \underset{(0.063)}{0.244}(g_t - \underset{(1.5)}{6.3}) + \varepsilon_{O,t}$$

$$r_t = \mu_{t-1} + \varepsilon_{r,t}$$

$$\Delta d_t = g_{t-1} + \varepsilon_{d,t}$$

$$\mu_t = (1 - \underset{(0.010)}{0.972})\underset{(4.2)}{12.5} + \underset{(0.010)}{0.972}\mu_{t-1} + \varepsilon_{\mu,t}$$

$$g_t = (1 - \underset{(0.005)}{0.991})\underset{(1.5)}{6.3} + \underset{(0.005)}{0.991}g_{t-1} + \varepsilon_{g,t}$$

with

$$Cov\left(\begin{bmatrix}\varepsilon_{\mu,t}\\ \varepsilon_{g,t}\end{bmatrix}\right) = \begin{bmatrix}\underset{(2.2)}{3.0} & \underset{(0.22)}{-0.32}\\ \underset{(0.22)}{-0.32} & \underset{(0.05)}{0.07}\end{bmatrix}.$$

estimation results

- The above equations further imply the following two equations:

$$\log G_t/O_t = const + \underset{(0.021)}{0.058}(\mu_t - \underset{(4.2)}{12.5}) - \underset{(0.068)}{0.016}(g_t - \underset{(1.5)}{6.3}) + \varepsilon_{G/O,t}$$

$$\log G_t \cdot O_t = const - \underset{(0.024)}{0.014}(\mu_t - \underset{(4.2)}{12.5}) + \underset{(0.112)}{0.472}(g_t - \underset{(1.5)}{6.3}) + \varepsilon_{G \cdot O,t}$$

omitted variable bias

Theorem

Suppose the price of asset i follows:

$$\log P_t^i = \alpha^i + \beta_\mu^i \mu_t + \beta_g^i g_t. \tag{17}$$

Then if we run $r_{t+1} = \mu_t + \varepsilon_{r,t+1}$ *onto* $\log P_t^i$ *in a univariate OLS regression, the slope coefficient is*

$$\hat{\beta}_{OLS} = \frac{1}{\beta_\mu^i} \frac{Var(\beta_\mu^i \mu_t) + Cov(\beta_\mu^i \mu_t, \beta_g^i g_t)}{Var(\log P_t^i)}. \tag{18}$$

Equivalently, the "omitted variable bias" (OVB) is $\frac{Var(\beta_g^i g_t) + Cov(\beta_\mu^i \mu_t, \beta_g^i g_t)}{Var(\log P_t^i)}$*. When* $Cov(\mu_t, g_t) = 0$*, the OVB reduces to the "attenuation bias":*

$$\hat{\beta}_{OLS} = \frac{1}{\beta_\mu^i} \frac{Var(\beta_\mu^i \mu_t)}{Var(\beta_\mu^i \mu_t) + Var(\beta_g^i g_t)}. \tag{19}$$

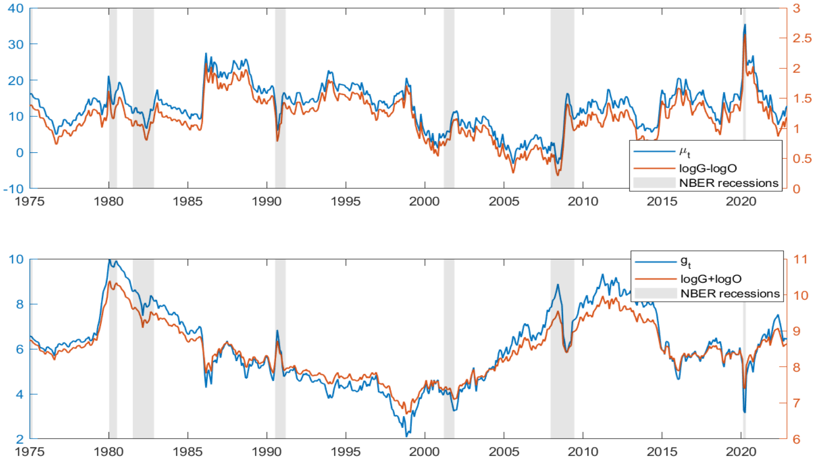
estimation results

Table 17: Variance Decomposition

Panel A			
	Dis rate (μ_t)	Div growth (g_t)	Covariance
$Var(\log G)$	48%	154%	-101%
$Var(\log O)$	7%	68%	26%
$Var(\log G/O)$	91%	1%	9%
$Var(\log G \cdot O)$	1%	113%	-14%
Panel B			
$Var(r)$	Expected (μ_t)	Unexpected ($\varepsilon_{r,t}$)	
100%	1.9%	98.1%	
$Var(\Delta d)$	Expected (g_t)	Unexpected ($\varepsilon_{d,t}$)	
100%	8.4%	91.6%	

- gold/oil prices are both mainly driven by expected div growth news, implying return predictive regression is strongly biased
- for gold, the OVB is $\frac{Var(\beta_g^G g_t)+Cov(\beta_\mu^G \mu_t, \beta_g^G g_t)}{Var(\log P_t^G)} = (1.54 - 1.01/2)/1 \approx 100\%$

filtered states



filtered states

- filtered states turn out to be good return/dividend growth predictors:

Table 18: Univariate Predictability Using Filtered States

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r_{t+i}^{f}\right)=\beta_0+\beta_\mu \mu_t+\epsilon_{t+h}$$

$$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+i}=\beta_0+\beta_g g_t+\epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				Returns				
β_μ	1.21	1.10	1.09	1.01	0.94	0.88	0.51	0.36
s.e.	[0.40]	[0.40]	[0.40]	[0.42]	[0.39]	[0.36]	[0.23]	[0.15]
R^2 (%)	1.58	2.65	4.11	7.35	9.47	11.11	13.36	12.11
				Dividend growths				
β_g	1.00	1.00	1.01	0.99	0.95	0.88	0.56	0.59
s.e.	[0.31]	[0.32]	[0.34]	[0.37]	[0.42]	[0.47]	[0.57]	[0.41]
R^2 (%)	5.23	5.47	5.67	5.92	5.80	5.33	4.02	9.85

predictive regressions under model-simulated data

- we simulate data from the model, run predictive regressions, and compare with the real data. results are similar

▸ data

Table 19: Univariate return predictability: model-simulated data

$$\frac{12}{h}\sum_{i=1}^{h} r_{t+i} = \beta_0 + \beta_X X_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G/O$				
$\beta_{\log G/O}$	15.83	15.55	15.28	14.50	13.79	13.10	9.05	6.48
s.e.	[5.40]	[5.40]	[5.35]	[5.30]	[5.25]	[5.22]	[5.05]	[4.74]
R^2 (%)	1.82	3.46	4.93	8.54	11.20	13.18	17.54	15.80
				$X = \log G \cdot O$				
$\beta_{\log G \cdot O}$	-4.98	-4.86	-4.74	-4.40	-4.10	-3.82	-2.27	-1.36
s.e.	[4.04]	[4.03]	[4.04]	[4.05]	[4.06]	[4.08]	[4.21]	[4.04]
R^2 (%)	0.66	1.25	1.79	3.16	4.24	5.12	8.85	10.54
				$X = \log G$				
$\beta_{\log G}$	3.92	3.94	3.98	3.98	3.98	3.96	3.61	3.12
s.e.	[11.4]	[11.4]	[11.4]	[11.4]	[11.4]	[11.3]	[10.9]	[10.5]
R^2 (%)	0.35	0.67	0.99	1.87	2.68	3.41	7.88	10.96
				$X = \log O$				
$\beta_{\log O}$	-10.59	-10.38	-10.16	-9.56	-9.00	-8.49	-5.52	-3.69
s.e.	[5.35]	[5.34]	[5.31]	[5.22]	[5.14]	[5.10]	[4.99]	[4.72]
R^2 (%)	1.20	2.26	3.22	5.56	7.30	8.62	12.19	12.07

predictive regressions under model-simulated data

data

Table 20: Univariate dividend growth predictability: model-simulated data

$$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+1} = \beta_0 + \beta_X X_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G/O$				
$\beta_{\log G/O}$	-2.78	-2.75	-2.73	-2.66	-2.60	-2.54	-2.09	-1.76
s.e.	[1.03]	[1.03]	[1.03]	[1.03]	[1.03]	[1.02]	[1.01]	[1.02]
R^2 (%)	3.46	6.33	8.75	14.17	17.79	20.25	25.30	24.09
				$X = \log G \cdot O$				
$\beta_{\log G \cdot O}$	2.33	2.31	2.29	2.23	2.17	2.11	1.71	1.39
s.e.	[0.44]	[0.44]	[0.44]	[0.44]	[0.45]	[0.45]	[0.49]	[0.53]
R^2 (%)	5.99	10.97	15.18	24.57	30.70	34.87	41.61	36.34
				$X = \log G$				
$\beta_{\log G}$	3.72	3.69	3.66	3.56	3.45	3.36	2.67	2.10
s.e.	[1.71]	[1.72]	[1.71]	[1.72]	[1.73]	[1.73]	[1.77]	[1.75]
R^2 (%)	3.18	5.78	7.97	12.78	15.86	17.97	21.80	19.78
				$X = \log O$				
$\beta_{\log O}$	3.22	3.20	3.17	3.09	3.01	2.94	2.39	1.96
s.e.	[0.61]	[0.60]	[0.60]	[0.60]	[0.61]	[0.62]	[0.66]	[0.72]
R^2 (%)	6.04	11.06	15.31	24.81	31.06	35.30	42.52	37.66

an extended model

- in the above model, we didn't consider the restrictions on model parameters imposed by the Campbell and Shiller (1988) identity and the price-dividend ratio
- Van Binsbergen and Koijen (2010) show that Campbell-Shiller present value calculation implies that, given the AR(1) dynamics for μ_t and g_t, the log price-dividend ratio should be a linear function in μ_t and g_t
- thus, we do not have two free state variables
- in the next, we consider an extended model that is desciplined by the price-dividend ratio

an extended model

- consider the model:

$$\begin{aligned}
\log G_t &= \alpha^G + \beta^G_\mu(\mu_t - \bar{\mu}) + \beta^G_\theta \theta_t + \beta^G_g(g_t - \bar{g}) + \varepsilon_{G,t} \\
\log O_t &= \alpha^O + \beta^O_\mu(\mu_t - \bar{\mu}) + \beta^O_\theta \theta_t + \beta^O_g(g_t - \bar{g}) + \varepsilon_{O,t} \\
r_t &= \mu_{t-1} + \theta_{t-1} + \varepsilon_{r,t} \\
\Delta d_t &= g_{t-1} + \varepsilon_{d,t} \\
\mu_t &= (1-\rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t} \\
\theta_t &= \rho_\theta \theta_{t-1} + \varepsilon_{\theta,t} \\
g_t &= (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \varepsilon_{g,t}
\end{aligned}$$

with

$$Cov\left(\begin{bmatrix} \varepsilon_{G,t} \\ \varepsilon_{O,t} \\ \varepsilon_{d,t} \end{bmatrix}\right) = \begin{bmatrix} \sigma^2_G & & \\ & \sigma^2_O & \\ & & \sigma^2_d \end{bmatrix}$$

$$Cov\left(\begin{bmatrix} \varepsilon_{\mu,t} \\ \varepsilon_{\theta,t} \\ \varepsilon_{g,t} \end{bmatrix}\right) = \begin{bmatrix} \sigma^2_\mu & \rho_{\mu,\theta}\sigma_\mu\sigma_\theta & \rho_{\mu,g}\sigma_\mu\sigma_g \\ \rho_{\mu,\theta}\sigma_\mu\sigma_\theta & \sigma^2_\theta & \rho_{\theta,g}\sigma_\theta\sigma_g \\ \rho_{\mu,g}\sigma_\mu\sigma_g & \rho_{\theta,g}\sigma_\theta\sigma_g & \sigma^2_g \end{bmatrix},$$

an extended model

- we interpret μ_t and θ_t as two separate components of discount rates, and g_t is still expected dividend growth
- given the literature evidence that PD is primarily driven by long-term discount rate news (Cochrane (2011), Van Binsbergen and Koijen (2010)), we add a new state variable θ_t to allow for greater flexibility
- iterating on the Campbell-Shiller identity $r_{t+1} = \Delta d_{t+1} - pd_t + \kappa_0 + \kappa_1 pd_{t+1}$ allows us to obtain

$$pd_t = \frac{\kappa_0}{1-\kappa_1} + E_t\Big[\sum_{j=0}^{\infty} \kappa_1^j \Delta d_{t+1+j}\Big] - E_t\Big[\sum_{j=0}^{\infty} \kappa_1^j r_{t+1+j}\Big],$$

  where κ_0 and κ_1 are log-linearization coefficients

an extended model

- we then substitute in state dynamics to express pd_t as a linear function in states:

$$pd_t = A - B_1(\mu_t - \bar{\mu}) - B_2\theta_t + B_3(g_t - \bar{g})$$

for

$$A = \frac{\kappa_0 - \bar{\mu} + \bar{g}}{1 - \kappa_1}$$
$$B_1 = \frac{1}{1 - \kappa_1 \rho_\mu}$$
$$B_2 = \frac{1}{1 - \kappa_1 \rho_\theta}$$
$$B_3 = \frac{1}{1 - \kappa_1 \rho_g}.$$

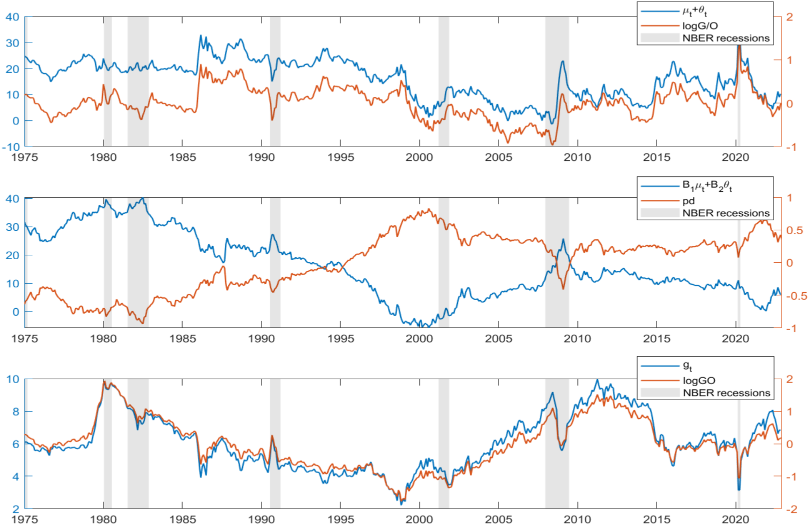
- our system eventually consists of two latent states (θ_t, g_t) and four observations $(\log G_t, \log O_t, \Delta d_t, pd_t)$
- we can still use kalman filter to write down the likelihood

an extended model

- we define $\mu_t + \theta_t$ as the "short-term discount rate" since it predicts next month's return
- we define $B_1\mu_t + B_2\theta_t$ as the "long-term discount rate" since from the Campbell-Shiller present value calculation:

$$pd_t = \frac{\kappa_0}{1-\kappa_1} + \underbrace{E_t\Big[\sum_{j=0}^{\infty}\kappa_1^j \Delta d_{t+1+j}\Big]}_{B_3 g_t} - \underbrace{E\Big[\sum_{j=0}^{\infty}\kappa_1^j r_{t+1+j}\Big]}_{B_1\mu_t + B_2\theta_t},$$

filtered states



filtered states

- filtered states turn out to be good return/dividend growth predictors:

Table 21: Univariate Predictability Using Filtered States

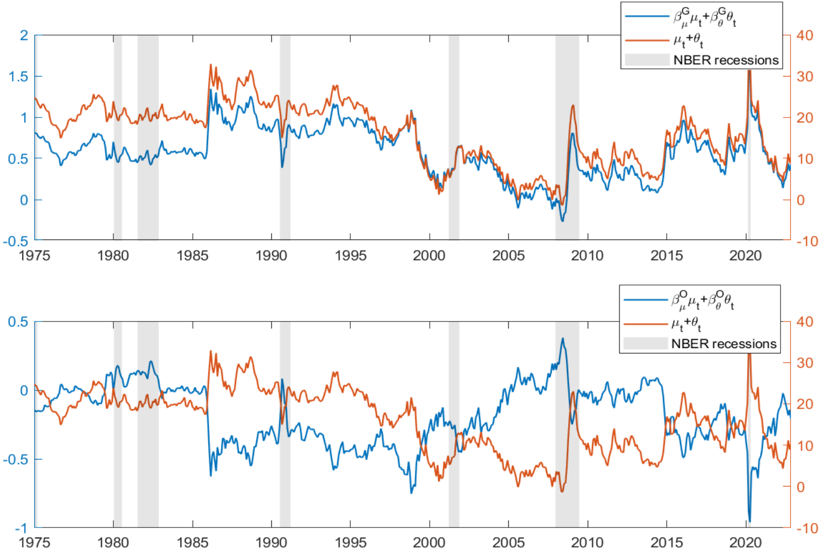
$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r_{t+i}^{f}\right)=\beta_0+\beta_{E^{Short}[r]}(\mu_t+\theta_t)+\epsilon_{t+h}$$

$$\frac{12}{h}\sum_{i=1}^{h}\left(r_{t+i}-r_{t+i}^{f}\right)=\beta_0+\beta_{E^{Long}[r]}(B_1\mu_t+B_2\theta_t)+\epsilon_{t+h}$$

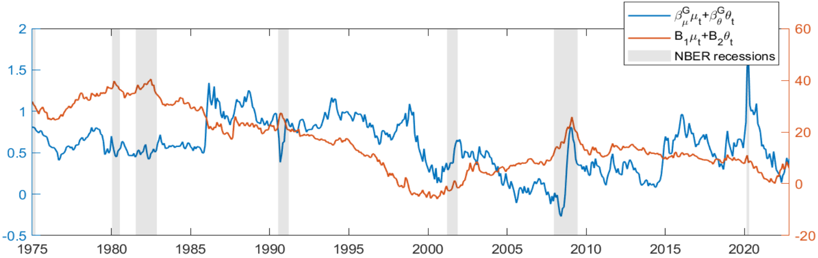
$$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+i}=\beta_0+\beta_g g_t+\epsilon_{t+h}$$

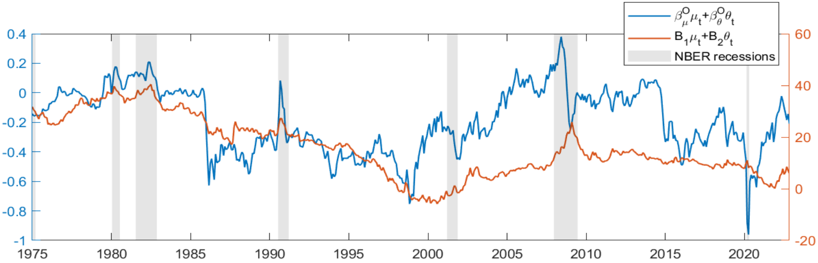
	1m	2m	3m	6m	9m	1y	3y	5y
			Returns by short-term discount rate					
$\beta_{E^{Short}[r]}$	0.88	0.80	0.79	0.74	0.71	0.66	0.30	0.22
s.e.	[0.28]	[0.28]	[0.28]	[0.30]	[0.28]	[0.27]	[0.19]	[0.13]
R^2 (%)	1.47	2.51	3.84	7.01	9.57	11.22	8.79	8.73
			Returns by long-term discount rate					
$\beta_{E^{Long}[r]}$	13.60	14.25	13.89	13.92	13.97	14.06	12.26	12.06
s.e.	[8.47]	[8.02]	[7.75]	[7.35]	[7.09]	[6.81]	[3.91]	[1.49]
R^2 (%)	0.26	0.96	1.58	3.67	5.72	8.01	25.74	44.33
			Dividend growths					
β_g	1.00	1.01	1.02	1.02	1.00	0.94	0.60	0.61
s.e.	[0.31]	[0.32]	[0.33]	[0.36]	[0.40]	[0.44]	[0.56]	[0.40]
R^2 (%)	5.39	5.74	6.03	6.53	6.59	6.22	4.86	11.09

how do commodity prices move with short-term discount rates?



how do commodity prices move with long-term discount rates?





estimation results

Table 22: Variance Decomposition

Panel A			
	Dis rate (μ_t, θ_t)	Div growth (g_t)	Covariance
$Var(\log G)$	103%	181%	-183%
$Var(\log O)$	28%	27%	45%
$Var(\log G/O)$	180%	28%	-108%
$Var(\log G \cdot O)$	8%	102%	-11%
$Var(pd)$	107%	2%	-9%
Panel B			
$Var(r)$	Expected $(\mu_t + \theta_t)$	Unexpected $(\varepsilon_{r,t})$	
100%	1.7%	98.3%	
$Var(\Delta d)$	Expected (g_t)	Unexpected $(\varepsilon_{d,t})$	
100%	4.9%	95.1%	

predictive regressions under model-simulated data

- we simulate data from the model, run predictive regressions, and compare with real data. results are similar

data

Table 23: Univariate return predictability: model-simulated data

$$\frac{12}{h}\sum_{i=1}^{h} r_{t+i} = \beta_0 + \beta_X X_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G/O$				
$\beta_{\log G/O}$	19.16	18.78	18.41	17.43	16.49	15.57	9.76	6.19
s.e.	[6.18]	[6.05]	[6.00]	[5.86]	[5.70]	[5.56]	[4.99]	[4.56]
R^2 (%)	1.68	3.22	4.63	8.19	10.90	12.88	16.19	13.36
				$X = \log G \cdot O$				
$\beta_{\log G \cdot O}$	-5.23	-5.02	-4.83	-4.26	-3.71	-3.19	-0.13	1.45
s.e.	[4.97]	[4.95]	[4.92]	[4.81]	[4.73]	[4.63]	[3.96]	[3.55]
R^2 (%)	0.40	0.75	1.07	1.83	2.36	2.74	4.36	7.31
				$X = \log G$				
$\beta_{\log G}$	8.15	8.23	8.29	8.53	8.78	8.97	9.65	9.31
s.e.	[9.16]	[9.14]	[9.20]	[9.15]	[9.01]	[8.92]	[8.23]	[7.61]
R^2 (%)	0.30	0.61	0.92	1.83	2.75	3.67	10.64	16.12
				$X = \log O$				
$\beta_{\log O}$	-13.89	-13.50	-13.15	-12.13	-11.16	-10.23	-4.55	-1.35
s.e.	[6.92]	[6.84]	[6.78]	[6.59]	[6.46]	[6.32]	[5.31]	[4.66]
R^2 (%)	1.02	1.93	2.74	4.68	6.00	6.85	6.64	5.69

predictive regressions under model-simulated data

▸ data

Table 24: Univariate dividend growth predictability: model-simulated data

$$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+1} = \beta_0 + \beta_X X_t + \epsilon_{t+h}$$

	1m	2m	3m	6m	9m	1y	3y	5y
				$X = \log G/O$				
$\beta_{\log G/O}$	-1.82	-1.79	-1.78	-1.73	-1.68	-1.63	-1.28	-1.03
s.e.	[1.03]	[1.03]	[1.03]	[1.02]	[1.02]	[1.00]	[0.95]	[0.89]
R^2 (%)	1.48	2.77	3.93	6.73	8.79	10.29	14.34	14.26
				$X = \log G \cdot O$				
$\beta_{\log G \cdot O}$	2.27	2.23	2.20	2.12	2.04	1.96	1.44	1.05
s.e.	[0.53]	[0.52]	[0.52]	[0.52]	[0.51]	[0.51]	[0.52]	[0.54]
R^2 (%)	4.15	7.73	10.85	18.08	23.03	26.39	30.73	24.94
				$X = \log G$				
$\beta_{\log G}$	3.04	3.00	2.95	2.81	2.69	2.58	1.79	1.21
s.e.	[1.35]	[1.35]	[1.36]	[1.37]	[1.37]	[1.37]	[1.38]	[1.41]
R^2 (%)	2.04	3.80	5.33	8.82	11.21	12.83	15.59	14.08
				$X = \log O$				
$\beta_{\log O}$	3.13	3.09	3.05	2.94	2.84	2.75	2.07	1.56
s.e.	[0.74]	[0.75]	[0.75]	[0.74]	[0.74]	[0.74]	[0.76]	[0.76]
R^2 (%)	3.97	7.40	10.40	17.41	22.29	25.67	30.82	25.82

PC predictive regressions

Table 25: PC predictive regressions

	1m	2m	3m	6m	9m	1y	3y	5y
$\frac{12}{h}\sum_{i=1}^{h}\Delta d_{t+i} = \beta_0 + \beta_1 PC(1)_t + \epsilon_{t+h}$								
β_1	2.23	2.26	2.29	2.34	2.32	2.25	1.38	1.19
s.e.	[0.56]	[0.59]	[0.61]	[0.66]	[0.72]	[0.79]	[1.08]	[0.82]
R^2 (%)	7.28	7.76	8.18	9.20	9.66	9.73	7.20	11.48
$\frac{12}{h}\sum_{i=1}^{h} r_{t+i} = \beta_0 + \beta_2 PC(2)_t + \epsilon_{t+h}$								
β_2	22.24	20.32	20.38	19.22	18.62	17.46	7.90	4.34
s.e.	[8.23]	[8.15]	[8.32]	[8.86]	[8.29]	[7.50]	[4.19]	[3.27]
R^2 (%)	1.41	2.28	3.55	6.44	9.04	10.67	7.43	4.22
$\frac{12}{h}\sum_{i=1}^{h} r_{t+i} = \beta_0 + \beta_5 PC(5)_t + \epsilon_{t+h}$								
β_5	19.80	20.69	20.56	23.79	22.74	24.06	27.48	22.56
s.e.	[15.71]	[13.78]	[12.75]	[11.75]	[12.40]	[12.12]	[8.99]	[5.62]
R^2 (%)	0.27	0.56	0.86	2.33	3.15	4.71	23.93	30.62

conclusion

- gold is a prominent hedging investment asset, provided the OVB related to expected economic fundamentals is addressed
- I use three approaches to show this
  - a state-space model quantifies the OVB to be nearly 100%
  - predictive regressions show that the gold-oil price ratio is a robust predictor of stock market returns, even at a one-day horizon
  - PCA further validates the state-space model: g_t and μ_t almost span the state-space of commodity pricing