Upside Risks and Models of Crude Oil Market

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Motivation

- Why oil
  - Oil price and volatility are strong economic state indicators
    - Historical recessions were associated with energy price surges
    - Oil prices went negative in 2020
- Why options
  - Options provide a hedge against price and volatility risks
- This paper studies
  - Economic implications of oil from option risk premiums
  - Empirical consistent option pricing models with a comprehensive consideration of risks featured in the energy market

What do we do

- Show asymmetric aversion to oil risk from option risk premiums
  - Empirical evidence from new data across energy sectors
    - monthly options on crude oil, natural gas, and heating oil
    - weekly options on crude oil
  - Consistent patterns from estimated model-based option risk premiums
- Provide a comprehensive framework to price commodities and options written on commodities
  - Model is featured by four risks: spanned, unspanned, jump, and **idiosyncratic risks**
  - Each risk corresponds to one empirical properties of commodities prices
  - The model is estimable by Kalman filtering
- New ingredients provided in the paper
  - New data to identify oil option risk premiums
  - New model for oil option risk premiums
  - New estimation strategies

Key findings

- Empirically, we show the aversion to large upside risks on oil returns, which is not reflected on the downside
- Analytically, we provide solutions for **oil price volatility**, **options prices** and **option risk premiums**
  - Each term exhibits state-dependence on idiosyncratic volatility
- The estimated model fits risk-neutral return volatility, option prices, VIX and upside option risk premiums from data
  - Both non-idiosyncratic and idiosyncratic volatility are relevant

Data fact 1: aversion to return upside in crude oil

- Weekly options
- Hedge on upside: negative premiums
- Hedge on downside: insignificant
- Hedge conditioning on high oil market uncertainty: significant negative on both sides

Panel A: Properties of excess returns of *weekly* options on oil futures (weekly, %)

	OTM puts		Straddle	OTM calls	
Moneyness	6%	3%		3%	6%
Option delta	-10	-25		25	10
σ^{Black} (%)	43.6	40.5	40.2	39.8	40.5
Unconditional Mean	**2**	**12**	**-2**	$-23^{\bullet}$	$-49^{\bullet}$
95% Bootstrap CI	$\lfloor -43, 40 \rfloor$	$\lfloor -18, 39 \rfloor$	$\lfloor -11, 6 \rfloor$	$\lfloor -45, -3 \rfloor$	$\lfloor -79, -25 \rfloor$
NW[*p*]	(0.91)	(0.44)	(0.61)	(0.03)	(0.00)
# positive (out of 345)	42	73	139	63	27

Data fact 1: aversion to return upside in crude oil

- Monthly options
- Hedge on upside: negative premiums
- Hedge on downside: insignificant
- Hedge conditioning on high oil market uncertainty: significant negative on both sides

Panel B: Properties of excess returns of *monthly* options on oil futures (monthly, %)

	OTM puts		Straddle	OTM calls	
Moneyness	10%	5%		5%	10%
Option delta	-10	-25		25	10
σ^{Black} (%)	37.7	35.8	34.5	34.4	35.5
Unconditional Mean	**-22**	**-8**	**-1**	**-6**	$-36^{\bullet}$
95% Bootstrap CI	$\lfloor -53, 6 \rfloor$	$\lfloor -31, 13 \rfloor$	$\lfloor -9, 7 \rfloor$	$\lfloor -30, 15 \rfloor$	$\lfloor -66, -11 \rfloor$
NW[*p*]	(0.16)	(0.48)	(0.84)	(0.60)	(0.03)
# positive (out of 360)	45	77	155	87	37
Conditional on $VIX_t^{SP\&500}$ being *Low*	-27	-6	1	-13	-44
Conditional on $VIX_t^{SP\&500}$ being *Medium*	-7	6	6	5	-32
Conditional on $VIX_t^{SP\&500}$ being *High*	-32	-24	$-10^{\bullet}$	-11	$-33^{\bullet}$
Conditional on VIX_t^{oil} being *Low*	5	24	$13^{\bullet}$	-4	-48
Conditional on VIX_t^{oil} being *Medium*	-22	-16	1	8	-14
Conditional on VIX_t^{oil} being *High*	$-48^{\bullet}$	$-33^{\bullet}$	$-16^{\bullet}$	$-24^{\bullet}$	$-47^{\bullet}$
Oil futures curve is in *backwardation*	$-61^{\bullet}$	$-36^{\bullet}$	-2	19	-11
Oil futures curve is in *contango*	11	15	0	$-27^{\bullet}$	$-57^{\bullet}$

Data fact 1: more evidence

- Same patterns in option returns across the energy sector
  - Monthly options on natural gas
  - Monthly options on heating oil

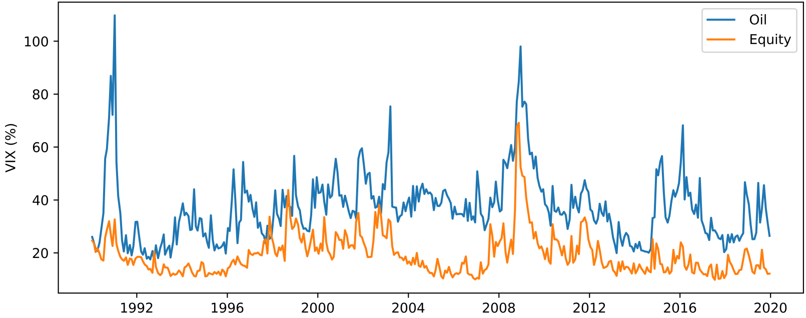
Panel A: Properties of excess returns of options on natural gas (Henry Hub, monthly (%))					
	OTM puts		Straddle	OTM calls	
Moneyness	10%	5%		5%	10%
Mean	5	5	-9	$-32^{\bullet}$	$-44^{\bullet}$
95% Bootstrap CI	⌊−33, 39⌋	⌊−22, 30⌋	⌊−19, 1⌋	⌊−56, −9⌋	⌊−73, −18⌋

Panel B: Properties of excess returns of options on heating oil (NY Harbor ULSD, monthly (%))					
	OTM puts		Straddle	OTM calls	
Moneyness	10%	5%		5%	10%
Mean	3	-1	-1	$-22^{\bullet}$	$-56^{\bullet}$
95% Bootstrap CI	⌊−60, 56⌋	⌊−38, 32⌋	⌊−12, 9⌋	⌊−46, −1⌋	⌊−82, −35⌋

Panel C: Properties of excess returns of options on WTI crude oil (monthly (%))					
	OTM puts		Straddle	OTM calls	
Moneyness	10%	5%		5%	10%
Mean	19	16	1	$-24^{\bullet}$	$-64^{\bullet}$
95% Bootstrap CI	⌊−48, 75⌋	⌊−25, 52⌋	⌊−11, 12⌋	⌊−50, −2⌋	⌊−82, −45⌋

Data fact 2: idiosyncratic risk evidenced by oil VIX

- Average $\mathrm{VIX}_t^{\text{oil}}$ ($\mathrm{VIX}_t^{\text{S\&P500}}$) is 36.9% (19.2%)
  - Significant amount of idiosyncratic volatility implied by a single index asset pricing model (e.g., CAPM)
- Imperfect correlation of 0.62 (0.34 in changes)
- Idiosyncratic risk is likely needed to fit (1) level and (2) time-variations of VIX in oil market



Comparison of model properties across published papers

- **Our estimated model**: There are idiosyncratic, jump, unspanned, and spanned risks. The extracted variables are (i) idiosyncratic variance and (ii) non-idiosyncratic variance.
- Model 1: Gibson and Schwartz (1990) (models of futures prices)
- Model 2: Schwartz (1997) (models of futures prices)
- Model 3: Hilliard and Reis (1998) (model of futures prices)
- Model 4: Casassus and Collin-Dufresne (2005) (models of futures prices)
- Model 5: Kogan, Livdan, and Yaron (2009)
- Model 6: Trolle and Schwartz (2009) (with Heath-Jarrow-Morton setup for cost of carry)
- Model 7: Chiang, Hughen, and Sagi (2015)
- Model 8: Christoffersen, Jacobs, and Li (2016) (discrete-time GARCH framework)
- Model 9: Heath (2019)
- Model 10: Equilibrium models (e.g., Routledge, Seppi, and Spatt (2000), Ready (2018), and Bornstein, Krusell, and Rebelo (2023)).
- Model 11: Crosby and Frau (2022) (with Heath-Jarrow-Morton setup for cost of carry)
- Model 12: Christoffersen, Jacobs, and Pan (2022) (extract state-price density from crude oil option prices)
- Model 13: Gao, Hitzemann, Shaliastovich, and Xu (2022) (macrofinance model with production and consumption)
- Model 14: Jacobs and Li (2023)

	Our paper	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Consider idiosyncratic risks	✓	no	no	no	no	no	no	no	no	no	no	no	*na*	no	no
Consider jump risks	✓	no	no	✓	no	no	no	no	✓	no	no	✓	*na*	no	no
Consider unspanned risks	✓	no	no	no	no	no	✓	✓	*na*	no	no	✓	*na*	no	no
Explore model fit to patterns of option risk premiums	✓	no	no	no	no	no	no	no	no	no	no	no	*na*	no	no
Explore model fit to $\{VIX_t^{oil}\}^2 = \mathbb{E}_t^{\mathbb{Q}}(\{-\frac{2}{\Delta}\}\log(\frac{F_{t+\Delta}^{T_F}}{F_t^{T_F}}))$	✓	no	no	no	no	no	no	no	no	no	no	no	no	no	no

Comparison with Trolle and Schwartz (2009)

- Trolle and Schwartz (2009) provides a commodity modeling framework, featured by unspanned risks
  - A stochastic volatility HJM-type commodity model
  - Focus on option prices and level of volatilities
- Incremental contribution of this paper
  - Idiosyncratic terms shapes volatility, option prices, and their risk premiums
  - Jump risks with weekly option data
  - Estimation of option risk premiums (this paper extends to modeling under the physical measure)

Model setup: the pricing kernel

- $\frac{dM_t}{M_{t-}} = -r_t\, dt + \mho[\mathrm{V}_t] \underbrace{d\mathbb{z}_t^{\mathbb{P}}}_{\text{spanned risks}} + \Pi[\mathrm{V}_t] \underbrace{d\mathbb{u}_t^{\mathbb{P}}}_{\text{unspanned risks}} + \{\underbrace{(e^{J_t} - 1)d\mathbb{N}_t^{\mathbb{P}}}_{\text{jump risks}} - \underbrace{\lambda_{\mathrm{J}}^{\mathbb{P}}\, \mathbb{E}_t^{\mathbb{P}}(e^{J_t} - 1)dt}_{\text{compensator}}\},$
  - $\mho[\mathrm{V}_t] = -\chi_{\mathrm{v}}^{*}\sqrt{\mathrm{V}_t} \quad \text{and} \quad \Pi[\mathrm{V}_t] = -\chi_{\mathrm{u}}^{*}\sqrt{\mathrm{V}_t},$
  - $\underbrace{d\mathbb{N}_t^{\mathbb{P}}}_{\text{Poisson jump}} = \begin{cases} 1 & \text{with probability } \lambda_{\mathrm{J}}^{\mathbb{P}}\, dt \\ 0 & \text{with probability } 1-\lambda_{\mathrm{J}}^{\mathbb{P}}\, dt, \end{cases} \quad \text{where } J_t \sim \underbrace{\mathrm{Normal}(\mu_{\mathrm{J}}^{\mathbb{P}}, \{\sigma_{\mathrm{J}}^{\mathbb{P}}\}^2)}_{\text{oil jump size distribution}}$

- $d\mathbb{z}_t^{\mathbb{P}}$: spanned risks
- $d\mathbb{u}_t^{\mathbb{P}}$: unspanned risks considering model market incompleteness
- Evolution of V_t is driven by both spanned and unspanned risks
- $d\mathbb{N}_t^{\mathbb{P}}$: Jump risks (a Poisson random variable) with intensity $\lambda_J^{\mathbb{P}}$ and size J_t
- Idiosyncratic risks are not in the pricing kernel

Model setup: spot oil prices

$$\frac{dS_t}{S_{t-}} = (\underbrace{\mathbf{y}_t}_{\text{cost of carry}} - \frac{1}{dt}\mathrm{cov}_t^{\mathbb{P}}(\frac{dM_t}{M_{t-}}, \frac{dS_t}{S_{t-}}))\,dt + \sqrt{\mathrm{V}_t}d\mathrm{z}_t^{\mathbb{P}} + \sqrt{\mathrm{I}_t}d\varepsilon_t^{\mathbb{P}} + \{(e^{J_t}-1)d\mathbb{N}_t^{\mathbb{P}} - \lambda_{\mathrm{J}}^{\mathbb{P}}\mathbb{E}_t^{\mathbb{P}}(e^{J_t}-1)dt\}$$

- Cost of carry captures the combined effect of interest rates, storage cost, and the convenience yield
- $d\mathrm{z}_t^{\mathbb{P}}$: spanned risks
- $d\mathbb{N}_t^{\mathbb{P}}$: Jump risks (a Poisson random variable) with intensity $\lambda_J^{\mathbb{P}}$ and size J_t
- $d\varepsilon_t^{\mathbb{P}}$: idiosyncratic risks

More on model features

- I_t (V_t), the idiosyncratic (non-idiosyncratic) diffusive variance, follows a stochastic volatility model

$$
\begin{aligned}
d\mathrm{V}_t &= (\theta_{\mathrm{V}}^{\mathbb{P}} - \kappa_{\mathrm{V}}^{\mathbb{P}} \mathrm{V}_t)\, dt + \sigma_{\mathrm{V}} \rho_{\mathrm{V}} \sqrt{\mathrm{V}_t} \underbrace{d\mathbb{z}_t^{\mathbb{P}}}_{\text{spanned risks}} + \sigma_{\mathrm{V}} \sqrt{1-\rho_{\mathrm{V}}^2} \sqrt{\mathrm{V}_t} \underbrace{d\mathbb{u}_t^{\mathbb{P}},}_{\text{unspanned risks}} \quad \text{and} \\
d\mathrm{I}_t &= (\theta_{\mathrm{I}}^{\mathbb{P}} - \kappa_{\mathrm{I}}^{\mathbb{P}} \mathrm{I}_t)\, dt + \sigma_{\mathrm{I}} \sqrt{\mathrm{I}_t}\, d\varepsilon_t^{\mathbb{P}}.
\end{aligned}
$$

- Compensation for diffusive risks

$$
d\mathbb{z}_t^{\mathbb{P}} = d\mathbb{z}_t^{\mathbb{Q}} + \mho[\mathrm{V}_t]\, dt, \qquad d\mathbb{u}_t^{\mathbb{P}} = d\mathbb{u}_t^{\mathbb{Q}} + \Pi[\mathrm{V}_t]\, dt \quad \text{and} \quad d\varepsilon_t^{\mathbb{P}} = d\varepsilon_t^{\mathbb{Q}}.
$$

- Compensation for jump risks

$$
\lambda_{\mathrm{J}}^{\mathbb{Q}} = \lambda_{\mathrm{J}}^{\mathbb{P}} + \underbrace{\{\lambda_{\mathrm{J}}^{\mathrm{rp}}\}^2}_{\text{jump intensity}}, \mu_{\mathrm{J}}^{\mathbb{Q}} = \mu_{\mathrm{J}}^{\mathbb{P}} \times \underbrace{\exp(\mu_{\mathrm{J}}^{\mathrm{rp}})}_{\text{(absolute) mean jump size}}, \{\sigma_{\mathrm{J}}^{\mathbb{Q}}\}^2 = \{\sigma_{\mathrm{J}}^{\mathbb{P}}\}^2 \times \underbrace{\exp(\sigma_{\mathrm{J}}^{\mathrm{rp}})}_{\text{variance of jump size}}
$$

- Cost of carry specification

$$
\underbrace{\mathbf{y}_t}_{\text{cost of carry}} = \delta_{\mathbf{y}}^{\circ} + \delta_{\mathrm{I}}^{*} \times \mathrm{I}_t + \delta_{\mathrm{V}}^{*} \times \mathrm{V}_t
$$

Analytical results

- Futures price is exponential affine in V_t and I_t
  - $\frac{F_t^{T_F}}{S_t} = \exp\left(\mathbb{a}_{\text{fut}}[\tau] + \mathbb{b}_{\text{fut}}[\tau] \times \mathrm{V}_t + \mathbb{c}_{\text{fut}}[\tau] \times \mathrm{I}_t\right), \qquad \text{with } \tau \equiv T_F - t.$

- Risk-neutral futures return variance is linear in V_t and I_t
  - $\log(1 + \mathbb{E}_t^{\mathbb{Q}}(\{\frac{F_{t+\Delta}^{T_F}}{F_t^{T_F}} - 1\}^2) = \mathbb{a}_{\text{vol}}[\Delta] + \mathbb{b}_{\text{vol}}[\Delta] \times \mathrm{V}_t + \mathbb{c}_{\text{vol}}[\Delta] \times \mathrm{I}_t$

- Expected integrated variance of futures returns under $\mathbb{P}$ is linear in $\mathbf{V}_t$ and $\mathbf{I}_t$
  - Futures return variance under $\mathbb{P}$: $\mathrm{rv}_{\{t \to t+\Delta\}}^{\text{futures},\mathbb{P}} = \mathbb{a}_{\text{rv}}^{\text{fut}}[\Delta] + \mathbb{b}_{\text{rv}}^{\text{fut}}[\Delta] \times \mathrm{V}_t + \mathbb{c}_{\text{rv}}^{\text{fut}}[\Delta] \times \mathrm{I}_t$
  - Spot return variance under $\mathbb{P}$: $\mathrm{rv}_{\{t \to t+\Delta\}}^{\text{spot},\mathbb{P}} = \mathbb{a}_{\text{rv}}^{\text{spot}}[\Delta] + \mathbb{b}_{\text{rv}}^{\text{spot}}[\Delta] \times \mathrm{V}_t + \mathbb{c}_{\text{rv}}^{\text{spot}}[\Delta] \times \mathrm{I}_t$

- Variance spreads exhibit a zero exposure to I_t
  - $\mathrm{VSPREAD}_{\{t \to t+\Delta\}} = \mathbb{a}_{\text{vspread}}[\Delta] + \mathbb{b}_{\text{vspread}}[\Delta] \times \mathrm{V}_t$

- Five measurement equations in Kalman filtering

Estimation

- Formulate model solutions in state-space form
- $\mathbf{Y}_t$ is constructed using data on options, futures, and spot price
- VSPREAD helps identify parameters governing non-idiosyncratic volatility
- Maximum-likelihood estimation via Kalman filtering
- VIX and option prices are left for cross-validating model performance

$$\underset{(5\times1)}{\mathbf{Y}_t} \equiv \underbrace{\begin{bmatrix} \log\left(\frac{F_t^{T_F}}{S_t}\right) \\ \log\left(1+\mathbb{E}_t^{\mathbb{Q}}\left(\left\{\frac{F_{t+\Delta}^{T_F}}{F_t^{T_F}}-1\right\}^2\right)\right) \\ \mathrm{rv}_{\{t\to t+\Delta\}}^{\mathrm{spot},\mathbb{P}} \\ \mathrm{rv}_{\{t\to t+\Delta\}}^{\mathrm{futures},\mathbb{P}} \\ \mathrm{VSPREAD}_{\{t\to t+\Delta\}} \end{bmatrix}}_{\text{all data in monthly units}} = \underbrace{\begin{bmatrix} \mathbb{a}_{\mathrm{fut}}[\tau] & \mathbb{b}_{\mathrm{fut}}[\tau] & \mathbb{c}_{\mathrm{fut}}[\tau] \\ \mathbb{a}_{\mathrm{vol}}[\Delta] & \mathbb{b}_{\mathrm{vol}}[\Delta] & \mathbb{c}_{\mathrm{vol}}[\Delta] \\ \mathbb{a}_{\mathrm{rv}}^{\mathrm{spot}}[\Delta] & \mathbb{b}_{\mathrm{rv}}^{\mathrm{spot}}[\Delta] & \mathbb{c}_{\mathrm{rv}}^{\mathrm{spot}}[\Delta] \\ \mathbb{a}_{\mathrm{rv}}^{\mathrm{fut}}[\Delta] & \mathbb{b}_{\mathrm{rv}}^{\mathrm{fut}}[\Delta] & \mathbb{c}_{\mathrm{rv}}^{\mathrm{fut}}[\Delta] \\ \mathbb{a}_{\mathrm{vspread}}[\Delta] & \mathbb{b}_{\mathrm{vspread}}[\Delta] & \boxed{0} \end{bmatrix}}_{\equiv\,\mathbb{C}[\Theta]} \underbrace{\begin{bmatrix} 1 \\ \mathrm{V}_t \\ \mathrm{I}_t \end{bmatrix}}_{\mathbf{X}_t} + \widetilde{\boldsymbol{\varepsilon}}_t, \qquad \widetilde{\boldsymbol{\varepsilon}}_t \sim \mathcal{N}(0,\boldsymbol{\Omega})$$

Analytical results (for cross-validation)

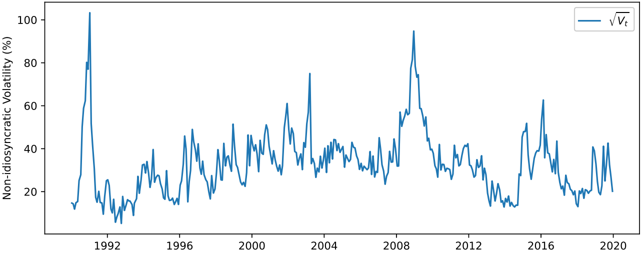
- $\{\mathrm{VIX}_t^{\mathsf{oil}}\}$ has an idiosyncratic variance component
  - $\{\mathrm{VIX}_t^{\mathsf{oil}}\}^2 = \mathbb{a}_{\mathsf{vix}}[\Delta] + \mathbb{b}_{\mathsf{vix}}[\Delta] \times \mathrm{V}_t + \mathbb{c}_{\mathsf{vix}}[\Delta] \times \mathrm{I}_t$
- Option prices and option risk premiums are solved in semi-analytic forms using Fourier inversion

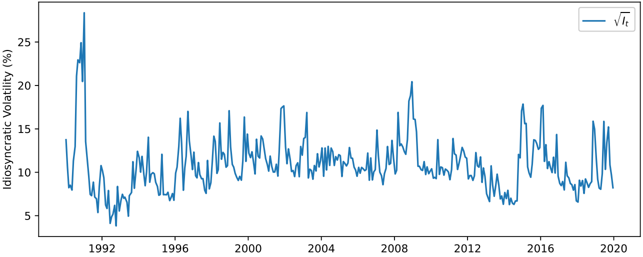
Estimation results

	Mean	SD	Min.	Percentiles 5th	25th	50th	75th	95th	Max.
Panel A: Features of extracted idiosyncratic variance and non-idiosyncratic variance									
$\sqrt{V_t}$ (annualized, %)	33	14	5	14	23	31	40	57	103
$\sqrt{I_t}$ (annualized, %)	11	3	4	7	9	10	12	17	28

	Panel B: Idiosyncratic variance			**Panel C: Jump component**					
	$\kappa_I^{\mathbb{P}}$	$\theta_I^{\mathbb{P}}$	σ_I	$\mu_J^{\mathbb{P}}$	$\sigma_J^{\mathbb{P}}$	$\lambda_J^{\mathbb{P}}$	μ_J^{rp}	σ_J^{rp}	λ_J^{rp}
Estimate	7.049	0.083	0.855	0.080	0.003	0.859	0.008	-0.004	0.131
(Standard Error)	(1.604)	(0.051)	(0.016)	(0.021)	(0.017)	(0.434)	(0.111)	(5.412)	(0.249)
	Panel D: Non-idiosyncratic variance			**Panel E: Risk premium components of variance**					
	$\kappa_V^{\mathbb{P}}$	$\theta_V^{\mathbb{P}}$	σ_V		χ_V^*	χ_u^*	ρ_V		
Estimate	3.432	0.339	0.722		3.337	-3.304	-0.677		
(Standard Error)	(0.000)	(0.033)	(0.021)		(0.947)	(1.006)	(0.000)		
	Panel F: Cost of carry			**Panel G: Measurement error**					
	δ_V°	δ_I^*	δ_V^*	$\sqrt{\Omega_1}$	$\sqrt{\Omega_2}$	$\sqrt{\Omega_3}$	$\sqrt{\Omega_4}$	$\sqrt{\Omega_5}$	$\mathbb{L}[\Theta]$
Estimate	0.010	0.049	0.074	0.021	0.000	0.008	0.014	0.008	5836
(Standard Error)	(0.017)	(0.525)	(0.080)	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)	

Extracted idiosyncratic and non-idiosyncratic variance





Summary picture of model fit and actual values

			Mean	SD	Min.	Percentiles 5th	25th	50th	75th	95th	Max.
1	VIX_t^{oil}	Model	37.4	13.2	16.3	21.2	28.5	35.6	43.7	60.0	106.9
		Actual	36.9	13.0	17.0	20.5	27.5	35.1	42.6	58.0	109.8
2	$\sqrt{\text{Variance}_t^{\mathbb{Q}}}$	Model	37.1	13.1	16.3	21.1	28.3	35.3	43.3	59.5	107.6
		Actual	37.1	13.1	16.3	21.1	28.3	35.3	43.3	59.5	107.6
3	$100 \times \frac{\text{10\% OTM put price}}{\text{futures price}}$	Model	1.1	1.1	0.0	0.2	0.5	0.8	1.3	2.7	9.9
		Actual	1.2	1.0	0.0	0.2	0.5	1.0	1.5	3.0	7.7
4	$100 \times \frac{\text{5\% OTM put price}}{\text{futures price}}$	Model	2.1	1.4	0.2	0.6	1.2	1.8	2.5	4.3	11.8
		Actual	2.2	1.3	0.2	0.6	1.3	2.0	2.8	4.4	9.4
5	$100 \times \frac{\text{ATM put price}}{\text{futures price}}$	Model	3.9	1.6	1.3	2.0	3.0	3.7	4.5	6.4	13.8
		Actual	4.2	1.5	1.1	2.2	3.1	4.0	4.9	6.7	11.3
6	$100 \times \frac{\text{ATM call price}}{\text{futures price}}$	Model	4.0	1.6	1.2	1.9	3.1	3.7	4.5	6.7	14.7
		Actual	4.2	1.6	1.4	2.2	3.2	4.0	4.9	6.9	12.2
7	$100 \times \frac{\text{5\% OTM call price}}{\text{futures price}}$	Model	2.1	1.5	0.2	0.5	1.2	1.8	2.6	4.5	12.5
		Actual	2.2	1.4	0.3	0.6	1.2	2.0	2.8	4.7	9.1
8	$100 \times \frac{\text{10\% OTM call price}}{\text{futures price}}$	Model	1.1	1.2	0.0	0.1	0.4	0.7	1.3	3.0	11.6
		Actual	1.1	1.1	0.0	0.2	0.4	0.8	1.4	2.9	7.9
9	Futures return volatility	Model	35.4	12.3	15.7	20.2	27.1	33.6	41.3	56.4	100.3
		Actual	32.2	15.1	9.8	15.0	22.7	29.2	37.7	61.4	133.7
10	Spot return volatility	Model	35.4	12.3	15.7	20.2	27.1	33.7	41.3	56.5	100.6
		Actual	34.9	18.4	11.7	15.9	24.1	30.9	39.6	71.4	176.4

Pricing error cross-validated by option prices

	Panel A: Dollar Option Pricing Errors (model - actual)			Panel B Percentage Option Pricing Errors ($100 \times \log(\frac{\text{model}}{\text{actual}})$)			Panel C: Regression $\log(\frac{\text{actual}_t}{\text{futures}_t}) = \alpha + \beta \log(\frac{\text{model}_t}{\text{futures}_t}) + \tilde{e}_t$				
	Average	Absolute	RMSE	Average	Absolute	RMSE	α	NW[p]	β	NW[p]	R^2
10% OTM puts	\$0.09	\$0.13	\$0.25	9	27	36	-0.05	0.83	0.97	0.00	83
5% OTM puts	\$0.15	\$0.19	\$0.34	10	19	25	-0.15	0.40	0.94	0.00	86
ATM puts	\$0.18	\$0.22	\$0.38	5	11	13	-0.06	0.60	0.96	0.00	88
ATM calls	\$0.06	\$0.11	\$0.22	6	10	13	-0.17	0.06	0.93	0.00	89
5% OTM calls	\$0.13	\$0.18	\$0.32	8	19	25	-0.34	0.02	0.89	0.00	87
10% OTM calls	\$0.18	\$0.22	\$0.38	2	27	36	-0.32	0.16	0.93	0.00	85

Panel D: Cross-sectional fit to option prices across moneyness

$100 \times \log(\frac{\text{model}}{\text{actual}})$	Mean	SD	Min	5	25	50	75	95	Max.
Average errors (all options)	7	21	-61	-33	-2	6	20	40	65
Absolute errors (all options)	19	14	0	3	8	17	27	43	65
RMSE (all options)	22	15	0	4	9	19	30	48	78

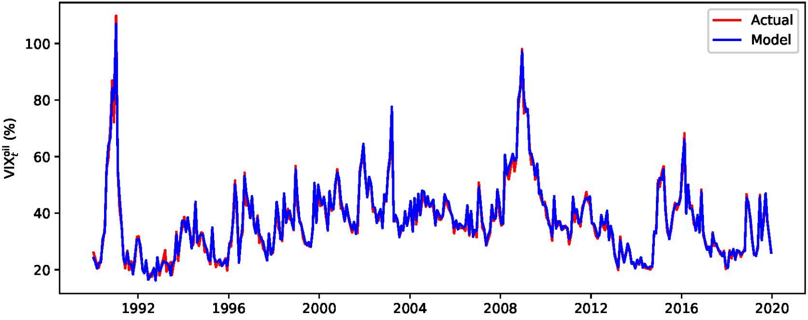
Information contents in option pricing errors

Panel E: $\log(\frac{\text{model}}{\text{actual}}) = \text{constant} + \sum_{j=1}^{3} \mathbb{b}_{\text{pc}}^{[j],\text{return}} \times \text{PC}_{t-1}^{[j],\text{return}} + \sum_{j=1}^{2} \mathbb{b}_{\text{pc}}^{[j],\text{variance}} \times \text{PC}_{t-1}^{[j],\text{variance}} + \tilde{e}_t$

	10% OTM put	5% OTM put	ATM put	ATM call	5% OTM call	10% OTM call
constant $\times$ 100	9	10	5	6	8	2
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.45)
$\overline{R}^2$ (%)	3	4	5	6	5	3

Pricing error cross-validated by VIX

- Absolute pricing error (RMSE) is 1.3 (1.7)
- $\log(\text{actual}_t) = \alpha + \beta \log(\text{model}_t) + \tilde{e}_t$
  - $\alpha = -0.04(0.00)$
  - $\beta = 0.98(0.00)$
  - $R^2 = 98\%$



Model-implied option risk premiums

- Risk Premium$^{\text{call}}_{\{t \to t+\Delta\}} = R^{\text{rf}}_{\{t \to t+\Delta\}} \left(\frac{\mathbb{E}^{\mathbb{P}}_t(\max(e^{\mathbb{x}} - e^k, 0))}{\mathbb{E}^{\mathbb{Q}}_t(\max(e^{\mathbb{x}} - e^k, 0))} - 1 \right)$
  - $\mathbb{x} \equiv \log\left(\frac{F^{T_F}_{t+\Delta}}{F^{T_F}_{t-}}\right)$ and $k = \log\left(\frac{K}{F^{T_F}_t}\right)$
- Within the range of bootstrapped average option returns
- The model is consistent with aversion to upside risk
- Idiosyncratic volatility affects option risk premiums by entering $\mathbb{P}$ and $\mathbb{Q}$ characteristic function (exponential affine in V_t and I_t)
- Put and call options on S&P 500 cannot have both negative risk premiums (e.g., Coval and Shumway (2001))

	10 OTM put (%)	Straddle (%)	10% OTM call (%)
Lower Bootstrap (data)	-53	-9	-66
Upper Bootstrap (data)	6	7	-11
Option Risk Premium (model) (mean)	-16	-1	-14

The role of idiosyncratic risks

- Spanning augmented with changes in idiosyncratic variance improve goodness-of-fit

	Panel A: Spanning regressions augmented with $\{I_{t+\Delta} - I_t\}$				
	OTM Puts		Straddle	OTM Calls	
	10	5		5	10
$\Lambda_{pc}^{[1],return}$	-8.0	-7.6	0.16	7.5	7.0
(NW[p])	(0.00)	(0.00)	(0.62)	(0.00)	(0.00)
$\Lambda_{pc}^{[2],return}$	1.57	3.94	-4.13	-19.4	-24.7
(NW[p])	(0.70)	(0.15)	(0.01)	(0.01)	(0.03)
$\Lambda_{pc}^{[3],return}$	11.8	5.7	-0.64	-21.2	-32.4
(NW[p])	(0.31)	(0.51)	(0.89)	(0.36)	(0.37)
Υ_{idio}	36.0	27.6	11.9	16.8	20.2
(NW[p])	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
constant$\times$100	-21	-7	0	-6	-36
(NW[p])	(0.08)	(0.30)	(0.89)	(0.47)	(0.00)
$\overline{R}^2$ (%)	36	52	21	48	34
$\overline{R}^2$ (restricted $\Upsilon_{idio} = 0$, %)	27	43	6	45	31
(when risks are spanned by oil futures)					

- Incorporated in option risk premiums with positive effects
- The non-idiosyncratic volatility is incorporated in option risk premiums with negative effects

Evidence on unspanned volatility risks and jumps

- $\rho_v = -0.677$: around half of the non-idiosyncratic volatility is unspanned
- $\lambda_J^{\mathbb{P}} = 0.859$: the expected number of jumps in oil prices per year is 10
- $\mu_J^{\mathbb{P}} = 8\%$: expected annualized jump size
- Contribution of jumps to spot return volatility $\frac{\lambda_J^{\mathbb{P}}(\{\mu_J^{\mathbb{P}}\}^2+\{\sigma_J^{\mathbb{P}}\}^2)}{\lambda_J^{\mathbb{P}}(\{\mu_J^{\mathbb{P}}\}^2+\{\sigma_J^{\mathbb{P}}\}^2)+V_t+I_t} = 7.1\%$

Conclusion

- A new model for oil price featuring (1) idiosyncratic, (2) jumps, (3) unspanned, (4) spanned risks
  - Model estimation supports relevance of risk components
  - Model is aligned with data
- Model fits oil VIX, option prices, and option risk premiums, while individual option prices are not used in estimation
- A general modeling framework for commodity derivatives