

# **New Tests of the Theory of Storage and the Theory of Normal Backwardation: Time and Frequency Dimensions**

Wenbin Cao<sup>a</sup>, Xiaoman Duan<sup>b</sup>, Scott Linn<sup>c</sup>,  
Pierre Six<sup>a</sup>

*<sup>a</sup>Finance Department, NEOMA Business School, France*

*<sup>b</sup>Department of General Business and Finance, Sam Houston State  
University, USA*

*<sup>c</sup>Division of Finance, University of Oklahoma, USA*

**J.P. Morgan Center for Commodities & Energy Management  
New Directions in Commodities Research Symposium  
August 2025**

# Our Focus

- Examination of the joint roles of time and frequency (decision horizon) in tests of the Theory of Storage and Normal Backwardation
- Highlight the complementary roles of both theories within the context of the oil futures market

# Some Motivation

- Why might frequency matter in the oil market?
  - Given the industrial nature of oil, the revenue streams associated with participation in the oil futures market will be tied to the horizons of market participants, such as producer-hedgers, speculators and traders, are naturally tied to expected market conditions and production and investment activity
  - Crude oil has distinct supply and demand dynamics and is subject to both originate from supply/demand shocks (Kilian, 2009; Kilian and Murphy, 2014; Baumeister and Kilian, 2016)
  - David (2019) links investors' habits to long-run decisions in the oil market, emphasizing the oil cycle's dual phases: exploitation (short-term production) and investment (long-term exploration and infrastructure).
  - Dew-Becker and Giglio (2016): Risk prices depend on investor preferences, not just consumption dynamics (offering a frequency-domain perspective that quantifies how investors value risks at different time horizons)
  - Constantinides and Ghosh (2017); Atanasov et al.(2020); Neuhierl and Varneskov (2021), Bandi and Tamoni (2022, 2023); Nagel and Xu (2023); Bessembinder et al. (2024); Karenkhe and de Roon (2024)
- Conjecture: If economic agents focus on a specific decision or planning horizon this 'Period' choice may impact the Basis or Risk Premium at that horizon

# Why should frequency matter in the oil market?

- Traditional frequency methods like Fourier analysis assume time series stationarity, decomposing time series into cycles spanning the entire sample
- Structural models tie futures and spot prices, and hence the basis, to the supply and demand of the commodity as well as its inventory (Williams and Wright, 1991; Pirrong, 2011).
- Theory of Storage and Theory of Normal Backwardation (hedging pressure) historically tested at specific frequencies (e.g. daily, monthly), which can miss frequency-specific information and result in mixed evidence (e.g., Rouwenhorst and Tang, 2012; Gorton et al., 2013) .
- Prior empirical studies largely examine these theories in isolation. Recent theoretical work (Acharya et al., 2013; Gorton et al., 2013; Ekeland et al., 2019) integrates these theories.

# Some Background

- **Theory of Storage** (see Kaldor, 1939; Working, 1949; Brennan, 1958) holders of inventories receive implicit benefits, the “**convenience yield**”, that declines as current inventory increases.
  - Refinements- optimal storage: Williams and Wright (1991); Pirrong (2011)
- **Theory of Normal Backwardation** (“hedging pressure” (Keynes (1930), Hicks (1939), Hirshleifer (1988, 1990))
- Commodity producers and inventory holders hedge future spot price risk by taking short positions in the futures market
  - Risk averse speculators require a ‘**risk premium**’ to induce them to provide hedging services
  - Refinements Ekeland (2019); DeRoos et al. (2000); Goldstein et al. (2014); Kang et al. (2020); Goldstein et al. (2022), +others
  - **Risk premium positively related to hedging pressure measured as (short – long); negatively related to (long – short)**
- **Models of jointly operating spot and futures markets** which yield conclusions about both the convenience yield and the futures risk premium (Gorton et al., 2013\*; Acharya et al., 2013; Ekeland et al., 2019)

# Primary Results

Basis is **negatively** related to inventory at intermediate and long frequencies (.75yr-10 year planning cycles), inventory **leads** and relation is supported across calendar time, but shift to Basis leading inventory in 2006 and after at shortest frequency

Futures risk premium is **negatively** related to hedging pressure (long – short) at longest frequencies (6-10 year planning cycles) and relation is supported across calendar time

Futures risk premium is **not consistently positively** related to hedging pressure (long – short) at the shortest Periods but exhibits transient episodes of a positive relation.

Additional results: A structural break in both the Basis – Inventory relation and the Risk Premium – Hedging Pressure relation occurred around 2005

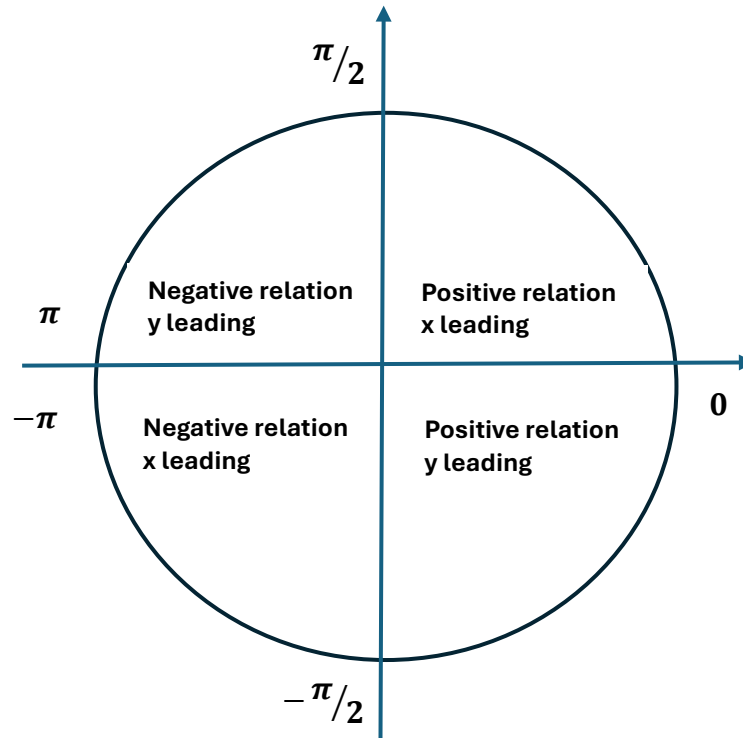
# Intuition: Wavelet Analysis

- Imagine you have a recording of a musical tune with different instruments playing at various times (a time series).
- Time-frequency analysis (Wavelet analysis) helps you pinpoint both what note (frequency and duration of frequency) is being played and its relative contribution, and when it occurs in calendar time.
- This is crucial for commodities, where price volatility can be driven by transient events (e.g., geopolitical shocks, supply disruptions) or persistent trends (e.g., inflation, technological changes).
- Wavelet analysis, two-dimensional analysis graphs
  - x-axis: Calendar time
  - y-axis: Period (Intuition in our context: If economic agents focus on a specific decision or planning horizon this 'Period' would impact the Basis or Risk Premium at that horizon)

# Wavelet Analysis

- **Continuous Wavelet Transform (Aguiar-Conraria and Soares, 2014)**
- **Partial Coherency (PCO)** reveals the partial effect of an explanatory variable on the dependent variable at a specific calendar time and frequency, accounting for control variables.
- The **Partial Phase Difference** quantifies the lead-lag relationship between two variables and whether the relationship is negative or positive, accounting for calendar time and frequency.
  - Each partial phase is computed for a specific frequency band. We present the partial phase alongside its 95% confidence interval.
- The magnitude of wavelet Partial Coherence indicates how strong the relation is between variables while the Phase provides information about the specific relation and synchronization between the variables (positive/negative and lead/lag)
- We use 5000 bootstraps to compute statistically significant regions (Berkowitz and Kilian, 2000)

# Partial Phase Interpretation (x, y)



## Partial Phase Difference Interpretation

Adapted from Aguiar-Conraria and Soares (2014)

# Basis and Inventory

- **Basis:**

- Measured as  $\left(\left(F_{1,t}/F_{2,t}\right) - 1\right) \times 365/30$
- $F_{1,t}$  is the nearest futures price at time t for the nearby contract, and  $F_{2,t}$  is the price of the next nearest maturing futures contract also at time t (follows for comparison Gorton et al., 2013).
- End of month, source: Bloomberg

- **Key variable of interest:**

- **Inventory**

- EIA; month-end normalized by 12-month moving average, effectively detrended (follows for comparison Gorton et al., 2013)

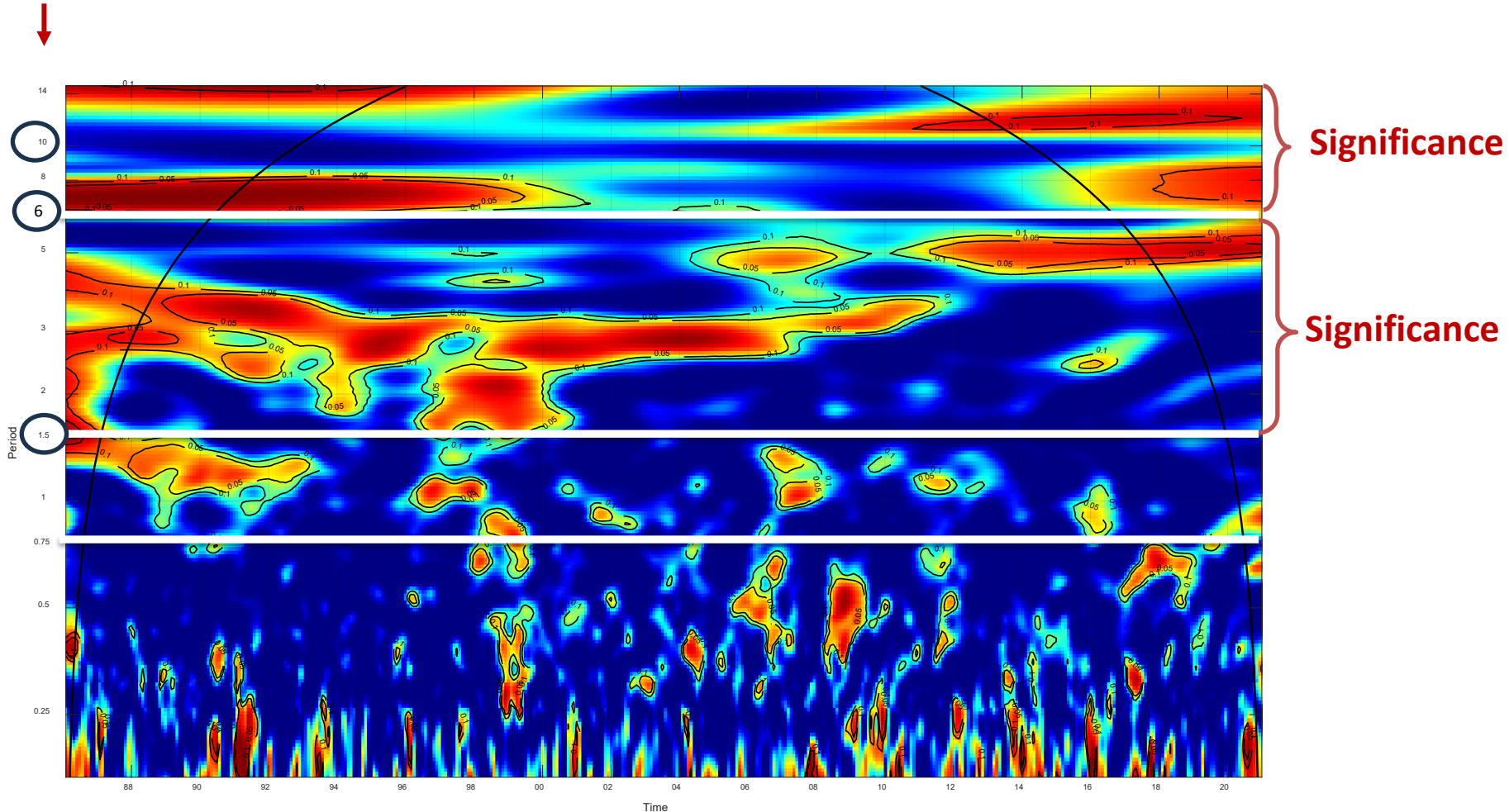
- **Controls:**

- Logarithm of crude oil price, end of month
- Monthly Volatility of crude oil price
- Hedging pressure (net long position of commercial traders (long – short) divided by total open interest, source U.S. CFTC) (follows for comparison Gorton et al., 2013)

# Basis and Inventory

Period Years

Partial Coherency (with controls)



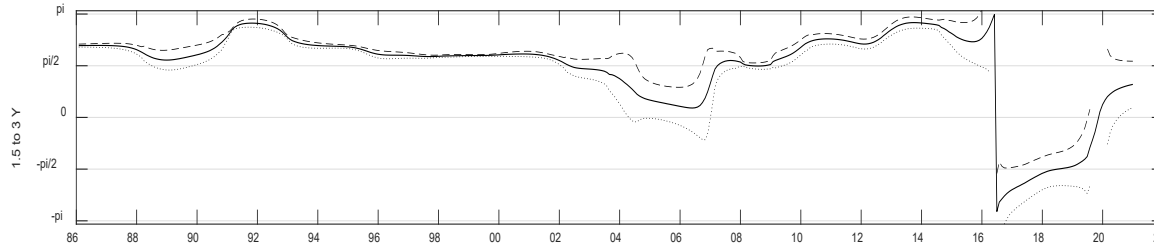
Red: Greater strength of the relation; Blue: less strength

# Basis and Inventory

## Partial Phase Difference (with controls, alternative Periods)

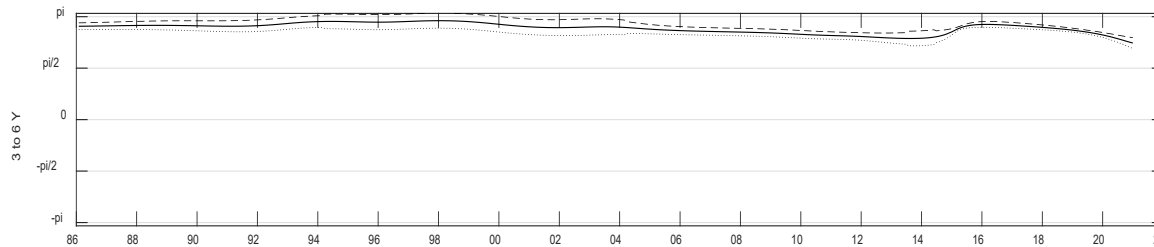
Period Years

Intermediate



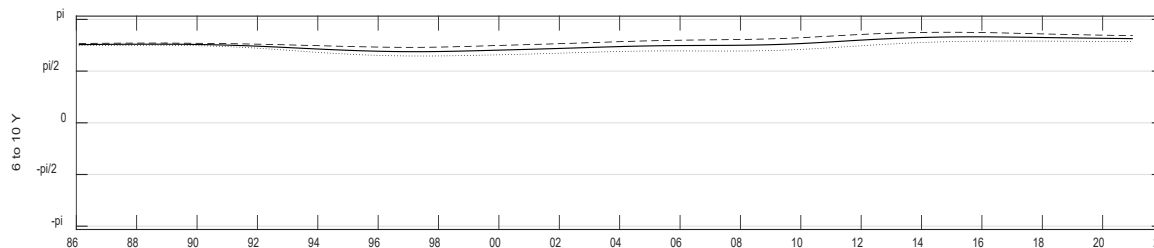
Inventory leads Basis, inverse relation but flips 2004-2008 and in 2016

Intermediate



Inventory leads Basis, inverse relation

Long



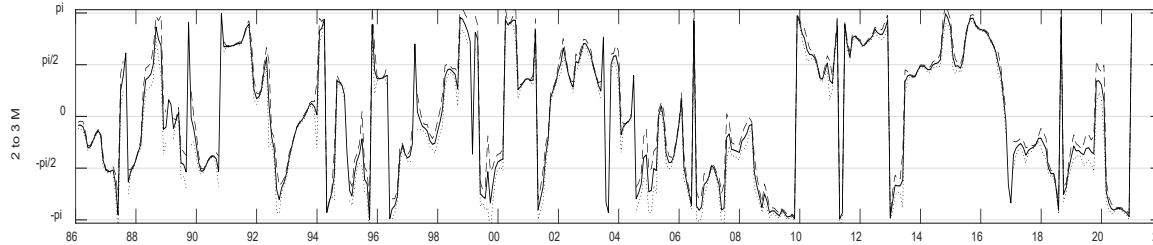
Inventory leads Basis, inverse relation

# Basis and Inventory

## Partial Phase Difference (with controls, alternative periods)

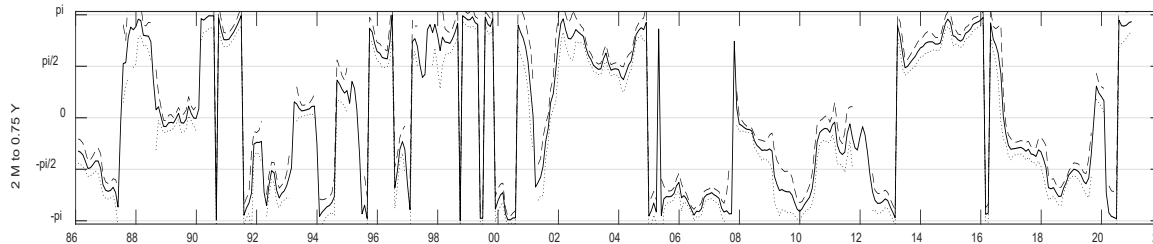
Period Years

Short



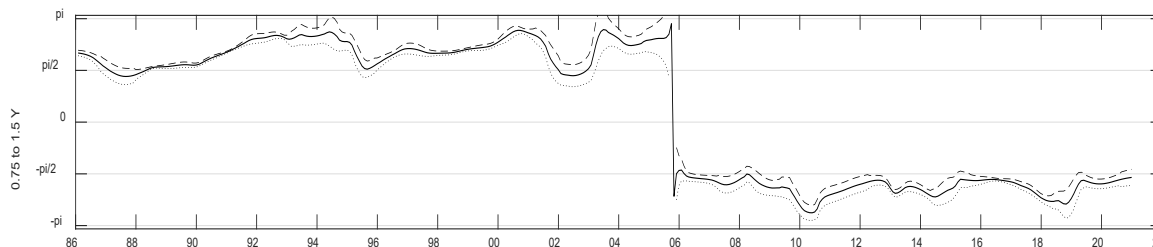
Mixed evidence

Short



Mixed evidence

Short



Inventory leads Basis, inverse relation, but shift to Basis leading inventory in 2006 and after with inverse relation flipping

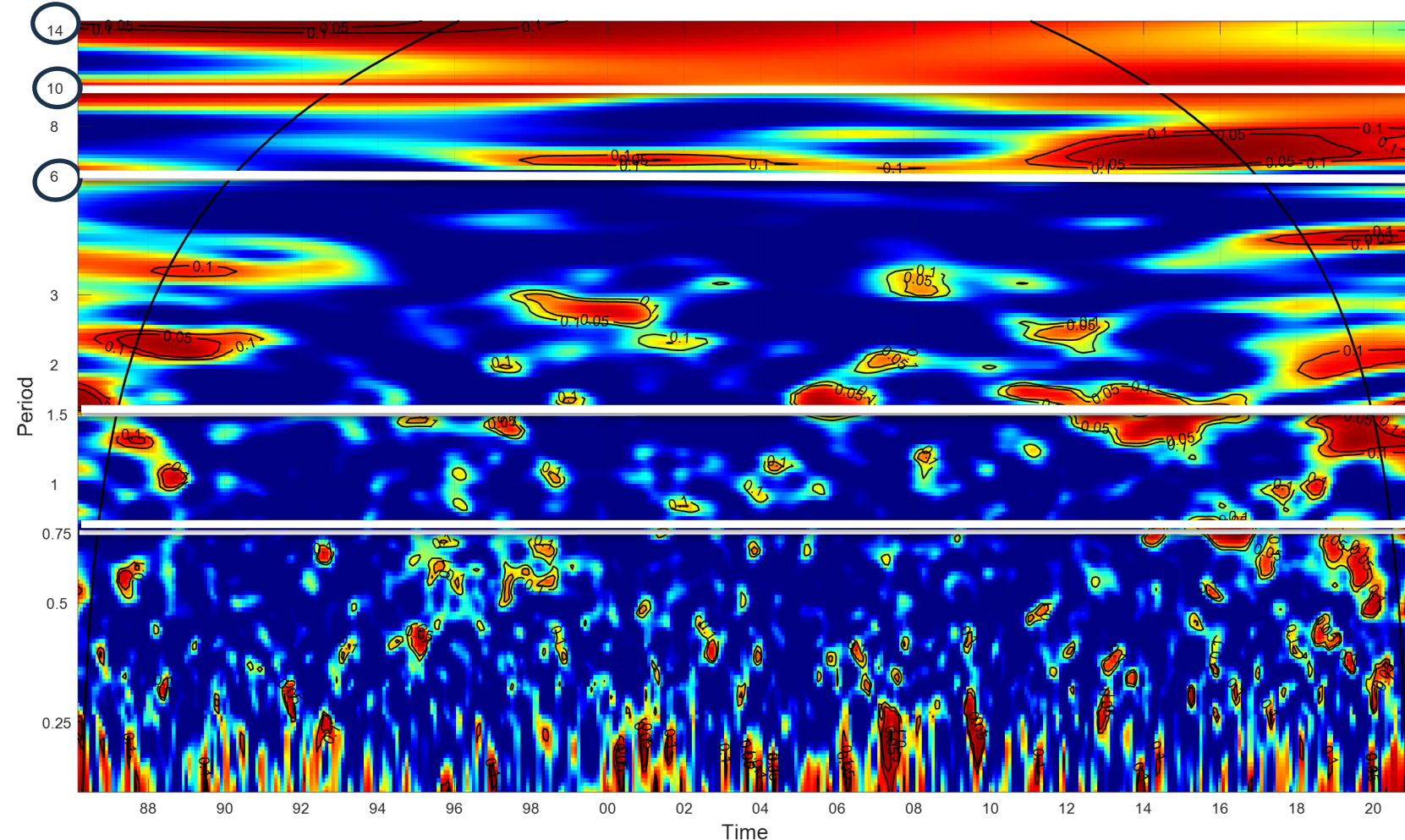
# Risk Premium and Hedging Pressure

- **Realized return:  $F_{1,t+1}/F_{1,t} - 1$ , with fixed maturity.**
  - To avoid the roll-over effect, if the nearest futures contract matures between  $[t, t+1]$ , we calculate the return using the next nearby futures contract
- **Key variable of interest:**
  - **Hedging pressure** (net long position of commercial traders (long – short) divided by total open interest, source U.S. CFTC) (follows for comparison Gorton et al., 2013)
- **Controls:**
  - Logarithm of crude oil price, end of month
  - Monthly Volatility of crude oil price
  - Net purchases of hedgers, changes in their net long positions divided by beginning open interest (follows Kang et al., 2020)
  - Inventory
    - EIA; month-end normalized by 12-month moving average, effectively detrended (follows for comparison Gorton et al., 2013)
  - S&P 500 risk premium, represented by realized S&P futures returns

# Risk Premium and Hedging Pressure

Period Years

Partial Coherency (with controls)



Strong significance

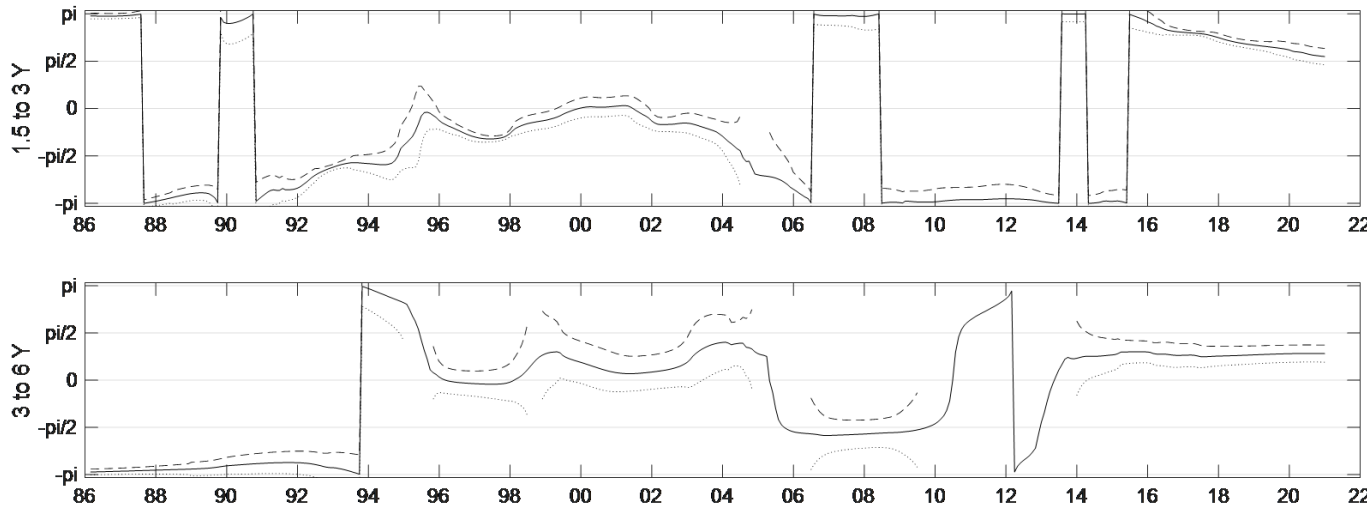
Red: Greater strength of the relation; Blue: less strength

# Risk Premium and Hedging Pressure

## Partial Phase Difference (with controls, alternative Periods)

Period Years

Intermediate



Mixed evidence,  
HP sometimes  
leads with  
inverse relation

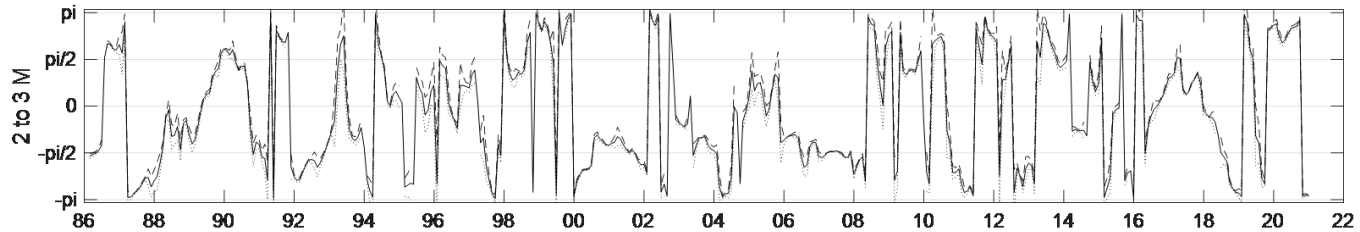
Hedging Pressure  
leads RP,  
inverse relation  
→ RP increasing in  
increased hedging

# Risk Premium and Hedging Pressure

Period Years

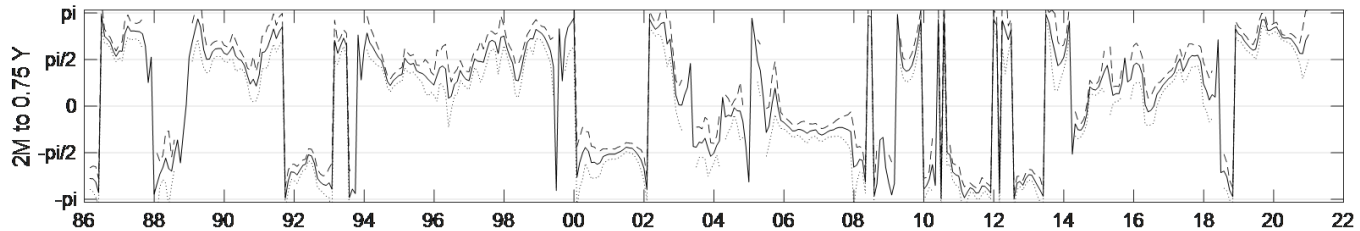
**Partial Phase Difference (with controls,  
alternative Periods)**

Short



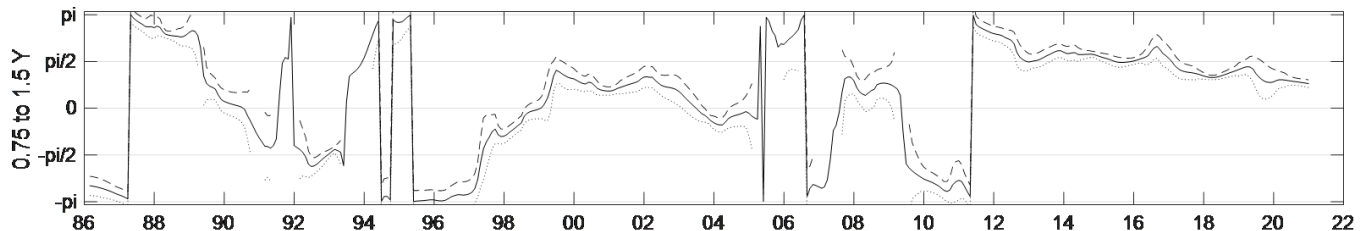
Mixed evidence

Short



Mixed evidence

Short



Mixed evidence

# Conclusion

- **Frequency matters** for the theory of storage as pertains to the oil futures market
- **Theory of Storage**
  - Basis is **negatively** related to inventory at intermediate and long frequencies (.75yr-10 year planning cycles), **inventory leads** and the relation is supported across calendar time, but **shifts** to **Basis leading inventory** in 2006 and after at shortest frequency
  - Strong support for the theory of storage at all calendar dates studied but not all Periods

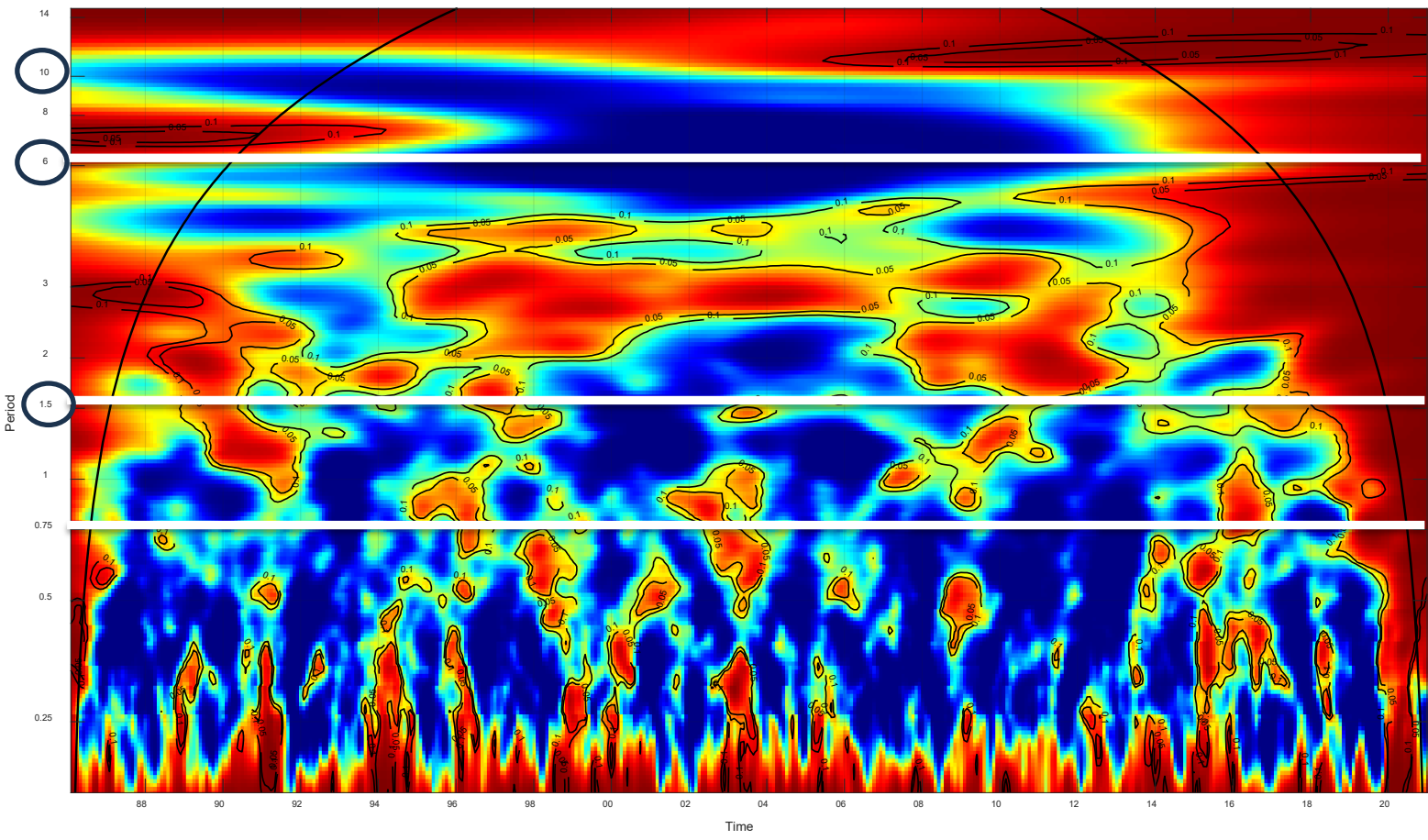
# Conclusion

- **Frequency matters** for the theory of hedging pressure (theory of normal backwardation) as pertains to the oil futures market
- **Theory of Hedging Pressure (Normal Backwardation)**
  - Futures risk premium is **negatively** related to hedging pressure (long – short) at longest frequencies (6-10 years planning cycles) and relation is supported across calendar time
  - Futures risk premium is **not consistently positively** related to hedging pressure (long – short) at the shortest Periods, but exhibits **transient episodes** of a positive relation.

# Supplemental Slides

# MCO: Basis and Inventory (with controls)

Years



Significance

Significance

**Red: Greater strength of the relation; Blue: less strength**

