

Hedging Pressure, Variance Risk Premia, and Expected Futures Returns in the Commodity Market

Asad Dossani¹ Sang Baum Kang² **Xuhui (Nick) Pan³**

¹Colorado State University

²Illinois Institute of Technology

³University of Oklahoma

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Motivation: Stochastic Variance in the Commodity Market

- Literature has used mean-variance preference to generate hedging or/and speculative demands of futures contracts.
 - Anderson and Danthine (1980, 1981), Bessembinder and Lemmon (2002)
 - Hong and Yogo (2012), Acharya, Lochstoer, and Ramadorai (2013), Wang (2021)
- Variance in the commodity market is stochastic and time varying.
 - Trolle and Schwartz (2009)
 - CBOE introduced the oil (gold) volatility index product OVX (GVZ) in 2008 (2011)

Motivation: Stochastic Variance in the Commodity Market

- This paper
 - Introduce a mean-variance model with stochastic variance in the commodity market.
 - Study the hedging and speculative demands for both futures and variance swap contracts.
 - Examine the relationship between futures risk premia (FRP) and variance risk premia (VRP).

Our Approach

- A reduced form model.
 - Commodity producers hedge by trading both futures and options to hedge.
 - Speculators supply liquidity.
 - Closed-form solutions for the relation between FRP and VRP.
- We empirically test the model implications using crude oil, gold, and copper.
 - Full period of Jan 1992 to Apr 2024.
 - Sub-sample period after 2009 when the options market becomes more liquid.

Main Results

- Our empirical results are largely consistent with the model implications across the commodities.
 - VRP are negative.
 - VRP negatively predicts futures returns.
 - The predictability is stronger when hedging pressure is high.
 - Results are more robust after 2009.

Related Literature

- Hedging or/and speculative demands of futures contracts.
 - Anderson and Danthine (1980, 1981); Hirshleifer and Subrahmanyam (1993); Bessembinder and Lemmon (2002); Hong and Yogo (2012); Acharya, Lochstoer, and Ramadorai (2013); Wang (2021)
- VRP in the commodity market.
 - Prokopczuk and Simen (2014); Ornelas and Mauad (2019)
- Predictability of VRP in various financial markets.
 - Bollerslev, Tauchen, and Zhou (2009); Bollerslev, Marrone, Xu, and Zhou (2014); Bekaert and Hoerova (2014); Bali and Zhou (2016); Heston and Todorov (2023)

Agents

Producers and speculators trade futures and options for risk sharing.

- Producers hedge exposure to price risk by taking short positions in futures contracts.
- Producers hedge exposure to variance risk by taking long positions in variance swap contracts (as a model result).
- Speculators provide liquidity and trade with producers.

Preference with Stochastic Variance

- Standard mean-variance (MV) preference

$$MV(W) \equiv E[W] - \frac{\gamma}{2} \text{var}[W]$$

- Producers

$$\begin{aligned} MVS V(W_{P_i}) &= E[W_{P_i}] - \frac{\gamma P_i}{2} \text{var}[W_{P_i} | \tilde{\sigma}^2 = \bar{\sigma}^2] \\ &\quad - \left[\frac{1}{\gamma P_i} \log M_{\tilde{\sigma}^2}(t_{P_i}) - \left(\frac{\gamma P_i}{2} \text{var}[W_{P_i} | \tilde{\sigma}^2 = \bar{\sigma}^2] - V_{\sigma}^{P_i} \bar{\sigma}^2 \right) \right] \end{aligned}$$

- Speculators

$$\begin{aligned} MVS V(W_{S_j}) &= E[W_{S_j}] - \frac{\gamma S_j}{2} \text{var}[W_{S_j} | \tilde{\sigma}^2 = \bar{\sigma}^2] \\ &\quad - \left[\frac{1}{\gamma S_j} \log M_{\tilde{\sigma}^2}(t_{S_j}) - \left(\frac{\gamma S_j}{2} \text{var}[W_{S_j} | \tilde{\sigma}^2 = \bar{\sigma}^2] - V_{\sigma}^{S_j} \bar{\sigma}^2 \right) \right] \end{aligned}$$

Equilibrium

- Both producers and speculators dislike stochastic variance.
- Optimal risk sharing (**both price risk and variance risk**) between hedgers and speculators leads to
 - Negative relationship between the futures risk premium and the VRP,

$$\mu - F = \gamma^* Y^* (\bar{\sigma}^2 - VRP),$$

where μ is the expected spot price, F the futures price, γ^* the aggregate risk aversion, Y^* the hedgeable endowment, and $\bar{\sigma}$ the expected variance.

Equilibrium

- In the presence of passive long position holders,

$$\mu - F = (Y^* - L) \gamma^* (\bar{\sigma}^2 - VRP),$$

where L denote the passive long holders position.

- If $(Y^* - L) > 0$, the futures returns are on average *positive* and VRP *negatively* predict the futures returns.
- If $(Y^* - L) < 0$, the futures returns are on average *negative* and VRP *positively* predict the futures returns.

Implications

- VRP in the commodity market are negative.
- VRP negatively predict futures returns.
- The predictability of futures returns depends on the interaction between VRP and hedging pressure.
- Results hold as long as agents are heterogeneous in either endowment, risk aversion, or residual basis risk.

Empirical Measurements

- VRP are defined as

$$VRP(t, \tau; T) = E_t^P [V(t, \tau; T)] - E_t^Q [V(t, \tau; T)]$$

- Horizon of the next 21 and 63 business days (VRP1 and VRP3)
- P -measure is estimated using the Mixed Data Sampling regression (MIDAS) (Ghysels, et al., 2019)
 - All forecasts are estimated out of sample without look-ahead bias.
- Q -measure is inferred from option prices (Bakshi and Madan, 2000; Bakshi, Kapadia, and Madan, 2003)

Data

- Daily futures and options data from the CME on crude oil, gold, and copper.
 - Jan 1992 to Apr 2024.
 - We focus on the most liquid month contracts.
- Daily futures and options data of other assets
 - S&P 500
 - 10-year Treasury Bond
 - USD/EUR exchange rate

Data

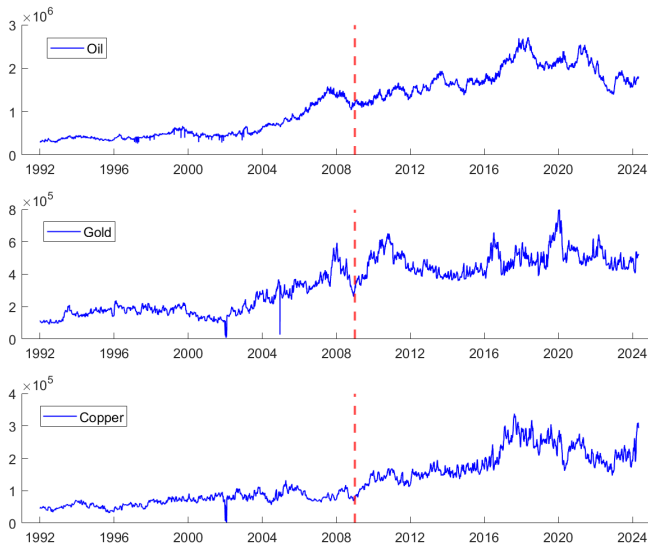
- Futures returns
 - A continuous return where contracts are rolled with 10 days to expiry
 - Another return series where contracts are rolled with 40 days to expiry
 - We predict futures returns on each roll date.

- Hedging pressure

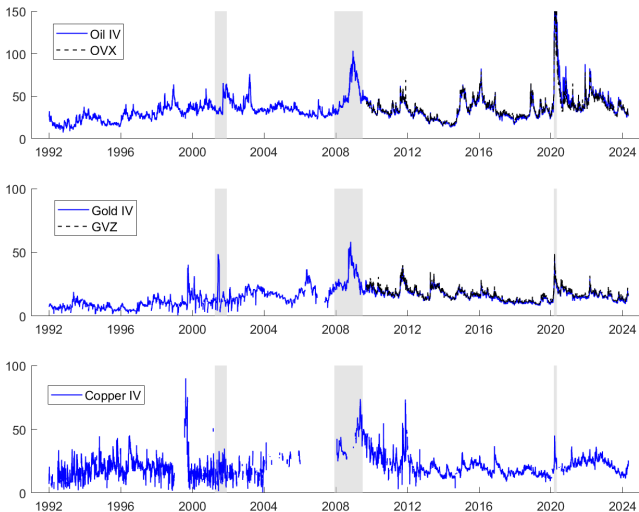
$$\text{CFTC pos}_t = \frac{(\# \text{ of short hedge pos})_t - (\# \text{ of long hedge pos})_t}{(\# \text{ of total hedge pos})_t}$$

- Measures the relative net short hedge positions to the total positions of large commercial traders
- Other predictors of futures returns.

Open Interests of Commodity Futures



Option-Implied Volatility of Commodity Futures



Predictive Regression

- We estimate the OLS regression:

$$Ret_t = \beta_0 + \beta_1(CFTC \cdot VRP)_{t-1} + \beta_2 VRP_{t-1} + \beta_3 CFTC_{t-1} + \beta_4' Y_{t-1} + \epsilon_t$$

where Ret is the futures return, VRP is the variance risk premium, $CFTC$ is the CFTC position, and Y is a vector of control variables.

- We estimate four regressions by varying the return measures (either Ret_{10} or Ret_{40}) and the VRP measures (VRP_1 or VRP_3).

Predictive Regression

Regression Results: Crude Oil, Jan 1992- Apr 2024

With Controls	V_1, R_{10}	V_1, R_{40}	V_3, R_{10}	V_3, R_{40}
Cons	0.002 (0.398)	0.001 (0.233)	-0.001 (-0.151)	-0.001 (-0.208)
VRP*CFTC	-0.026** (-2.847)	-0.019** (-2.163)	-0.027*** (-2.805)	-0.021** (-2.370)
VRP	0.033** (2.635)	0.022 (1.850)	0.026 (1.905)	0.016 (1.355)
CFTC	-0.002 (-0.381)	-0.001 (-0.214)	-0.000 (-0.074)	-0.000 (-0.029)
HistRet	-0.046 (-0.209)	0.073 (0.338)	-0.007 (-0.031)	0.103 (0.490)
OI	-0.202 (-0.638)	0.535** (2.472)	-0.132 (-0.411)	0.574*** (2.629)
Basis	-0.030 (-0.782)	-0.035 (-1.016)	-0.035 (-0.829)	-0.039 (-1.026)
Stor	0.000 (0.094)	0.000 (0.316)	-0.000 (-0.145)	0.000 (0.123)
Adj R ²	0.028	0.029	0.028	0.034
obs	387	387	387	387

Predictive Regression

Regression Results: Crude Oil, Jan 2009 - Apr 2024

With Controls	V_1, R_{10}	V_1, R_{40}	V_3, R_{10}	V_3, R_{40}
Cons	-0.005 (-0.431)	0.007 (0.694)	0.000 (0.037)	0.009 (0.874)
VRP*CFTC	-0.057*** (-2.702)	-0.029 (-1.613)	-0.049*** (-2.669)	-0.033** (-2.096)
VRP	0.078*** (2.847)	0.033 (1.520)	0.053** (2.037)	0.025 (1.226)
CFTC	0.000 (0.018)	-0.008 (-0.717)	-0.004 (-0.335)	-0.009 (-0.827)
HistRet	-0.108 (-0.293)	0.022 (0.063)	-0.071 (-0.213)	0.021 (0.064)
OI	-0.571 (-1.221)	0.589* (1.881)	-0.421 (-0.938)	0.648** (2.041)
Basis	0.019 (0.276)	-0.028 (-0.435)	-0.011 (-0.151)	-0.049 (-0.664)
Stor	-0.000 (-0.402)	-0.000 (-0.241)	-0.000 (-0.607)	-0.000 (-0.452)
Adj R ²	0.017	-0.005	0.016	0.013
obs	183	183	183	183

Predictive Regression

Regression Results: Gold, Jan 1992- Apr 2024

With Controls	V_1, R_{10}	V_1, R_{40}	V_3, R_{10}	V_3, R_{40}
Cons	0.004 (0.541)	0.004 (0.414)	0.007 (0.988)	0.003 (0.333)
VRP*CTFC	-0.018** (-2.591)	-0.012 (-1.103)	-0.001 (-0.195)	-0.008 (-0.874)
VRP	0.006 (1.502)	0.009 (1.476)	-0.002 (-0.297)	0.007 (1.050)
CTFC	0.009 (1.589)	0.017** (2.044)	0.005 (0.888)	0.016 (1.887)
HistRet	0.039 (0.162)	-0.312 (-0.988)	0.097 (0.393)	-0.290 (-0.843)
OI	-0.035 (-0.460)	-0.072 (-0.701)	-0.010 (-0.119)	-0.082 (-0.791)
Basis	0.048 (0.175)	0.013 (0.268)	0.075 (0.243)	0.006 (0.110)
Adj R ²	0.013	0.015	-0.014	0.007
obs	193	193	193	193

Predictive Regression

Regression Results: Gold, Jan 2009 - Apr 2024

With Controls	V_1, R_{10}	V_1, R_{40}	V_3, R_{10}	V_3, R_{40}
Cons	0.012 (1.327)	0.008 (0.684)	0.014 (1.425)	0.003 (0.267)
VRP*CTFC	0.006 (0.467)	-0.019* (-1.697)	0.011 (0.960)	-0.015 (-1.339)
VRP	-0.001 (-0.126)	0.009 (0.909)	-0.006 (-0.543)	0.001 (0.067)
CTFC	-0.025** (-2.078)	-0.003 (-0.183)	-0.026* (-1.941)	-0.000 (-0.018)
HistRet	0.635** (2.101)	0.112 (0.292)	0.637* (1.932)	-0.032 (-0.082)
OI	-0.137 (-1.079)	-0.172 (-1.077)	-0.148 (-1.170)	-0.196 (-1.243)
Basis	-0.480 (-0.995)	-0.020 (-0.362)	-0.491 (-0.958)	-0.045 (-0.791)
Adj R ²	-0.021	-0.001	-0.018	0.014
obs	91	91	91	91

Predictive Regression

Regression Results: Copper, Jan 1992- Apr 2024

With Controls	V_1, R_{10}	V_1, R_{40}	V_3, R_{10}	V_3, R_{40}
Cons	0.008 (0.873)	0.007 (0.686)	0.009 (0.927)	0.007 (0.769)
VRP*CFTC	0.009* (1.961)	-0.004 (-0.477)	-0.014 (-1.034)	-0.032*** (-2.810)
VRP	-0.006 (-1.134)	0.008 (0.581)	-0.000 (-0.008)	-0.006 (-0.384)
CFTC	0.011 (0.882)	0.007 (0.577)	0.008 (0.677)	0.008 (0.796)
HistRet	-0.118 (-0.615)	-0.010 (-0.061)	-0.103 (-0.528)	0.002 (0.009)
OI	0.206 (0.969)	0.113 (0.598)	0.210 (0.988)	0.052 (0.298)
Basis	0.066 (0.604)	0.012 (0.571)	0.083 (0.764)	0.007 (0.366)
Adj R ²	-0.011	-0.026	-0.010	0.036
obs	160	161	160	161

Predictive Regression

Regression Results: [Copper, Jan 2009 - Apr 2024](#)

With Controls	V_1, R_{10}	V_1, R_{40}	V_3, R_{10}	V_3, R_{40}
Cons	0.013 (0.747)	-0.003 (-0.182)	0.011 (0.606)	-0.001 (-0.049)
VRP*CFTC	-0.017 (-1.146)	-0.043*** (-3.526)	-0.024* (-1.895)	-0.042*** (-2.689)
VRP	-0.030* (-1.991)	-0.046** (-2.265)	-0.022 (-1.301)	-0.041** (-1.983)
CFTC	0.001 (0.049)	0.010 (0.465)	-0.006 (-0.279)	0.014 (0.684)
HistRet	-0.361 (-1.052)	-0.290 (-0.811)	-0.269 (-0.795)	-0.443 (-1.226)
OI	-0.240 (-0.783)	-0.209 (-0.670)	-0.214 (-0.649)	-0.211 (-0.645)
Basis	-0.354 (-1.292)	-0.133 (-1.602)	-0.330 (-1.290)	-0.109 (-1.416)
Adj R ²	0.013	0.112	-0.013	0.107
obs	75	76	75	76

Conclusion

- We study theoretically and test empirically the equilibrium relation between commodity VRP and futures returns.
- Our empirical evidence is largely consistent with model predictions, especially after 2009.
 - VRP are negative.
 - Negative relationship between VRP and futures returns in the absence of passive investors.
 - Predictability of futures returns varies with hedging pressure in the presence of passive investors.