A weak latent trend hides strong price predictability: an empirical method for an unrecognized problem.

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Angus Deaton (*Journal of Economic Perspectives* 2010) concludes from a mapping of current price to price next period, citing the example of cotton price:

"In theory, a country might save when prices are high against times when prices are low, but commodity prices are highly positively autocorrelated, so strategies to smooth government or household expenditure would often require periods of saving and dis-saving that are too long to be within the borrowing or lending capacity or indeed the political patience of the countries involved." (pp. 7–10).

Do the data support this hypothesis?
Let's consider samples of prices of cotton and also maize:

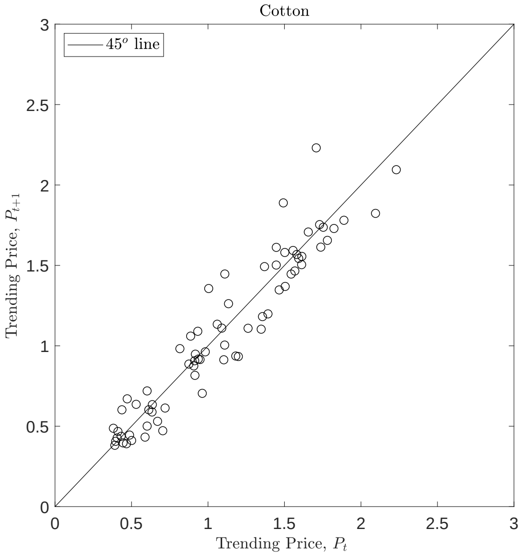


Figure: Annual cotton price realizations at t and t+1. 1936-2007

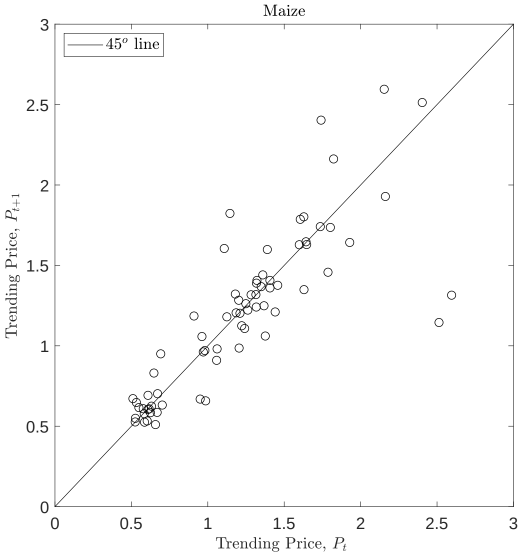


Figure: Annual maize price realizations at t and $t{+}1$. 1936-2007

Deaton is clearly right:

Choose any price level in these samples and the subsequent price seems to show little systematic tendency to, on average, move away from that price, whether it is low or high. Any average difference of next period prices from the 45 degree line is small relative to typical random deviations:

Angus Deaton and Guy Laroque (*Journal of Development Economics*, 2003), for many annual commodity price series:

"over the long run, the trend is small relative to the variability" (Deaton and Laroque JDE 2003 p. 291).

That is, the contribution of the secular trend movement to annual price variation, given any base price level in the sample, is small. Relative one year ahead spreads tell the same story:

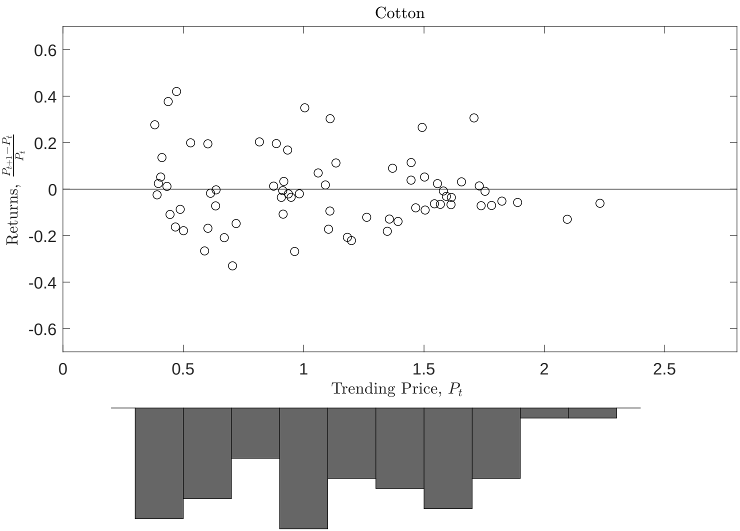


Figure: Cotton. Annual proportional price changes between t and $t+1$ mapped from price at t. 1936-2007

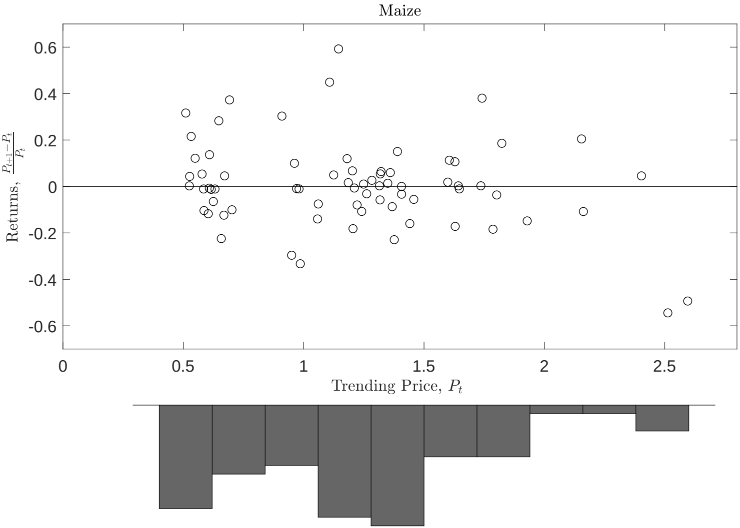


Figure: Maize. Annual proportional price changes between t and $t+1$ mapped from price at t. 1936-2007

A naive preliminary:

Despite the evidence reviewed thus far, even Deaton and Laroque 2003 admit that:

"it is possible to see upward or downward 'trends' over prolonged periods." (p. 291).

This is especially true for cotton, where one can make a case for an initial upward deterministic trend, then after a break a downward secular trend as in the Prebisch-Singer literature:

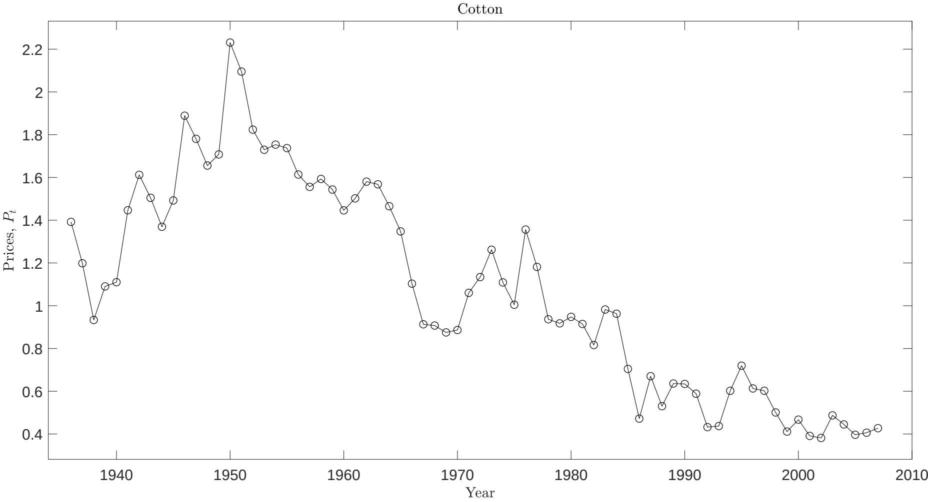


Figure: Annual real prices for cotton. 1936-2007

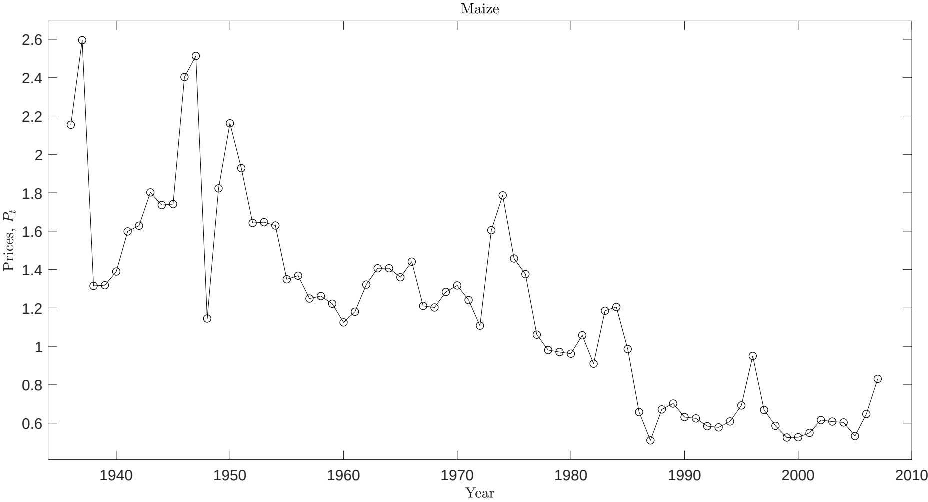


Figure: Annual real prices for maize. 1936-2007

Approaches to empirical challenges posed by trends:

(i) Ignore the possibility of a secular trend,
(ii) Use prior detrending, and fit model to detrended data even if trends in different variables are are not consistent (Roberts and Schlenker)
(iii) Assume linear or log-linear Euler equations,
(iv) Ignore the possible interaction of a trend and economic incentives in the Euler equation, or
(v) Assume independent errors.

Let's first try preliminary estimation and removal of an exponential trend in the data (as in Bobenrieth et al. AJAE 2020):

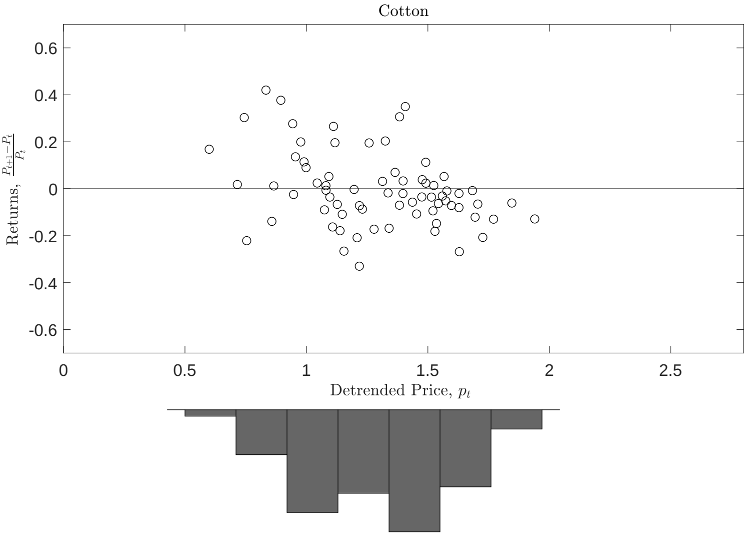


Figure: Preliminary Detrending. Annual proportional price changes between t and $t+1$ mapped from detrended price. 1936-2007

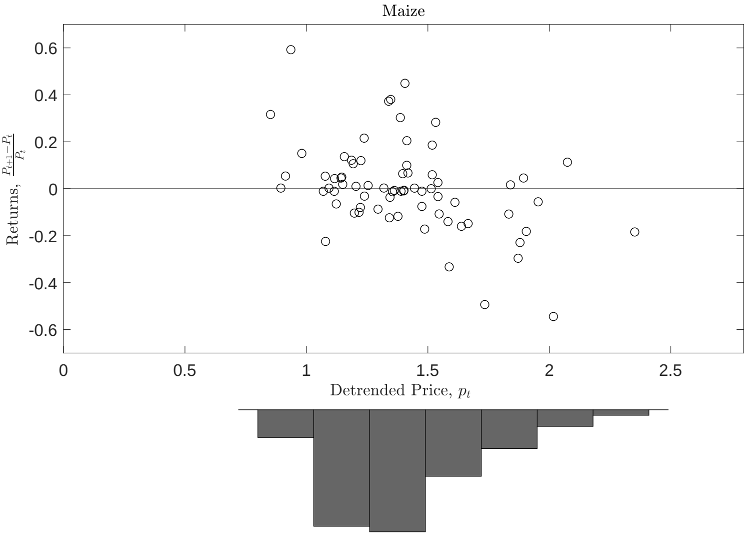


Figure: Preliminary Detrending. Annual proportional price changes between t and $t+1$ mapped from detrended price. 1936-2007

The results show promise, but there are problems with prior detrending:

(i) Trend might interact with behavioral variables.

(ii) How do you identify and handle apparent trend breaks?

(iii) Could different trends (e.g. stochastic and deterministic) appear in different regimes?

(iv)Trends in different variables (in a more complex empirical model) might be inconsistent, making detrending problematic

(v) There is no known method for calculating asymptotic standard errors if prices are generated from a storage model. (Bobenrieth et al. *American Journal of Agricultural Economics 2021*).

Is it possible to estimate all the parameters on trending data in a single step?

Elements of a model of speculation with random supply, non-negative stocks storable with the only cost the rate of interest on the value of stocks.:

Inverse consumption demand is:

$$P_t = F(C_t)$$

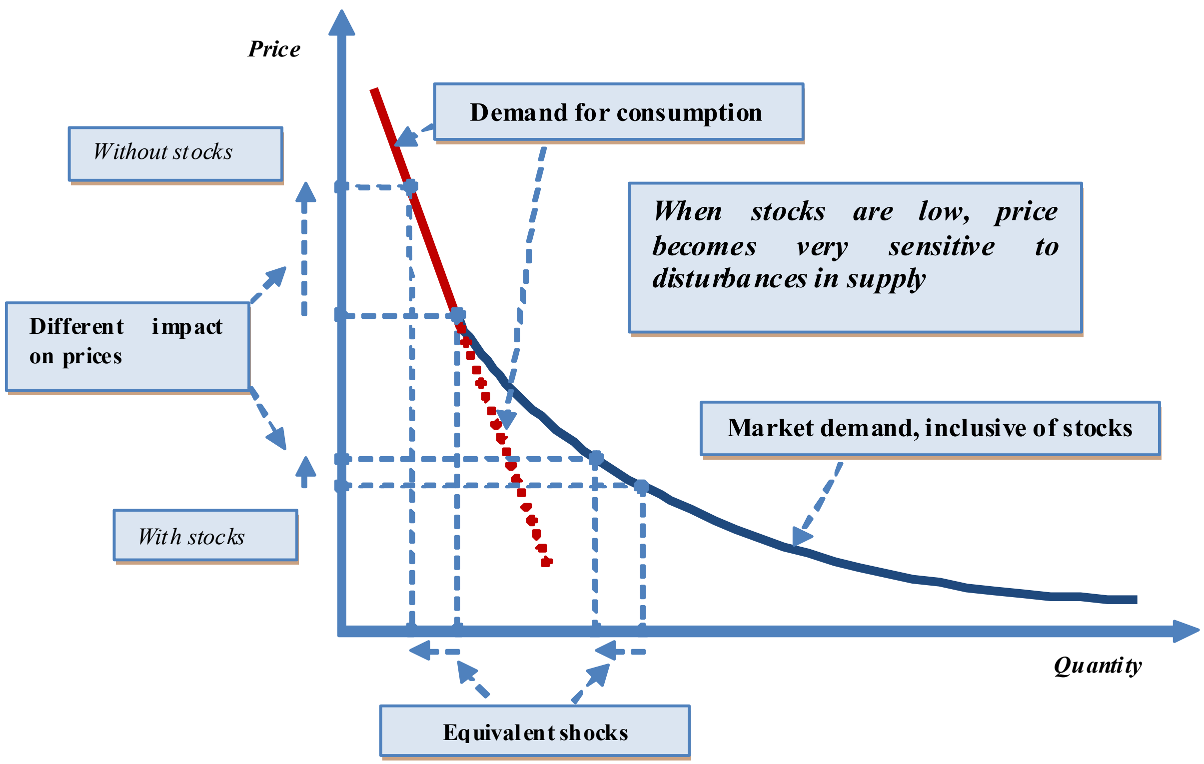
where C_t is consumption and P_t is observed real (trending) price. The real interest rate is r.

We assume an exponential trend less than unity in price of λ:

$$P_t = \lambda^t p_t$$

where p_t is "detrended" price with dynamics still affected by the trend.

Price is more sensitive to shocks when stocks are minimal.



Demand F is from the HARA class of utility functions:

$$\frac{F''F}{(F')^2} = \kappa$$

We assume:

an exogenous secular (upward) trend in production of the commodity, and

κ such that they are jointly consistent with an exponential trend in pric

In terms of observed prices, the model implies a price autoregression with a trending threshold $\lambda^t p^*$:

$$E_t P_{t+1} = \gamma \min[\lambda^t p^*, P_t]. \tag{1}$$

where $\gamma = 1 + r$

The non-stationary model maps to a latent stationary model, with "detrended" price p_t such that: $P_t = \lambda^t p_t = \lambda^t p(z_t)$, where p is a Stationary Rational Expectations price function, a function of "detrended" or "normalized" available supply z_t of the commodity.

Theorem: *The Markov process of normalized available supply $\Phi \equiv \{z_t\}_{t \geq 0}$ is uniformly ergodic, that is, it has a unique invariant probability measure ν_∞ which is a global attractor, and there exists constants $k > 1$ and $R < \infty$ such that for all initial z_0 we have:*

$$||\nu_t - \nu_\infty|| \leq Rk^{-t},$$

where $||\cdot||$ denotes the total variation norm, and ν_t is the distribution on z_t conditional on z_0.

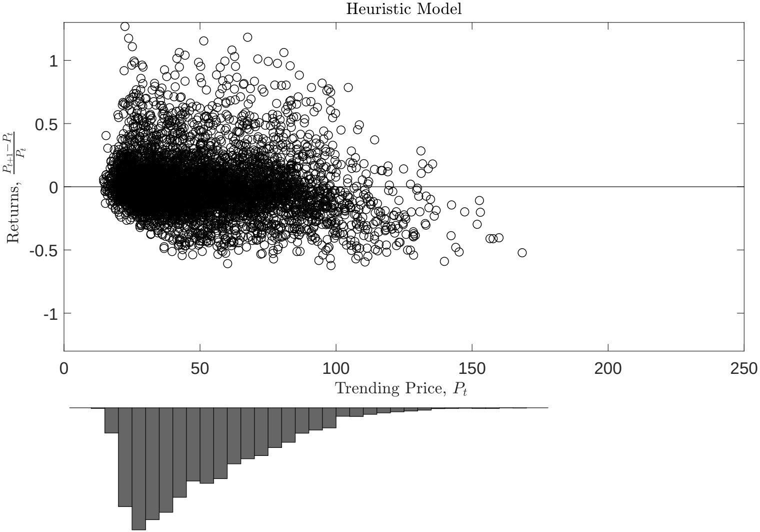


Figure: MonteCarlo: proportional price changes

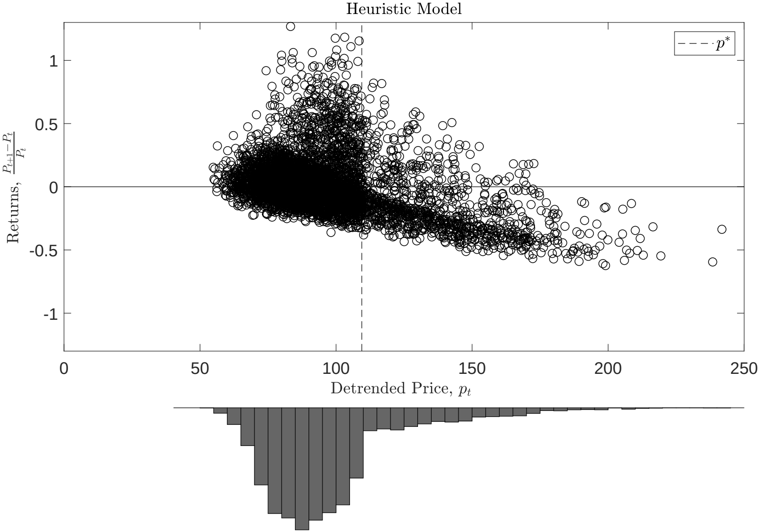


Figure: MonteCarlo: proportional price changes

Empirical estimation:

We use one-step nonlinear LS.
We normalize the regression model to address an identification problem:

$$Y_{t+1} \;=\; f_t(\theta_0) \;+\; \epsilon_{t+1} \tag{2}$$

where the true parameter vector is θ_0 *and* $\quad Y_{t+1} = \frac{P_{t+1}}{P_t}$

$$f_t(\theta) \equiv \gamma \min\left\{\frac{\lambda^t p^*}{P_t},\ 1\right\} .$$

This can be rewritten to highlight a dichotomy that occurs at $\lambda = \lambda_0$:

$$f_t(\theta) \equiv \gamma \min\left\{\frac{\lambda^t p^*}{P_t},\ 1\right\} = \gamma \min\left\{\left(\frac{\lambda}{\lambda_0}\right)^t \frac{p^*}{p_t},\ 1\right\}.$$

This dichotomy means we had to provide a new method of proof of consistency of the estimator.

Theorem. $\hat{\theta}_T$ *is strongly consistent, that is,*

$$\lim_{T\to\infty} ||\hat{\theta}_T \; - \; \theta_0|| \; = \; 0, \qquad a.s.$$

We also prove:

Theorem. $\left\{ T^{3/2}(\widehat{\lambda}_T - \lambda_0), \, T^{1/2}(\widehat{p^*}_T - p_0^*), \, T^{1/2}(\widehat{\gamma}_T - \gamma_0) \right\}_{T\in\mathbb{N}}$ *converges in distribution to a normal random vector with mean zero and covariance matrix* $\Sigma^{-1}\Lambda\Sigma^{-1}$.

$$\Lambda \equiv 2 \begin{pmatrix} \frac{2Ap_0^{*2}\gamma_0^2}{3\lambda_0^2} & \frac{Ap_0^*\gamma_0^2}{\lambda_0} & \frac{Ap_0^{*2}\gamma_0}{\lambda_0} \\ \frac{Ap_0^*\gamma_0^2}{\lambda_0} & 2A\gamma_0^2 & 2Ap_0^*\gamma_0 \\ \frac{Ap_0^{*2}\gamma_0}{\lambda_0} & 2Ap_0^*\gamma_0 & 2(B + Ap_0^{*2}) \end{pmatrix},$$

$$\Sigma \equiv \begin{pmatrix} \frac{2Cp_0^{*2}\gamma_0^2}{3\lambda_0^2} & \frac{Cp_0^*\gamma_0^2}{\lambda_0} & \frac{Cp_0^{*2}\gamma_0}{\lambda_0} \\ \frac{Cp_0^*\gamma_0^2}{\lambda_0} & 2C\gamma_0^2 & 2Cp_0^*\gamma_0 \\ \frac{Cp_0^{*2}\gamma_0}{\lambda_0} & 2Cp_0^*\gamma_0 & 2D \end{pmatrix},$$

$$A \equiv \lim_{t\to\infty} E\left(\frac{\epsilon_{t+1}}{p_t} 1_{\{p_t > p_0^*\}}\right)^2, \quad B \equiv \lim_{t\to\infty} E\left(\epsilon_{t+1} 1_{\{p_t \le p_0^*\}}\right)^2,$$

$$C \equiv \lim_{t\to\infty} E\left(\tfrac{1}{p_t} 1_{\{p_t > p_0^*\}}\right)^2, \; D \equiv \lim_{t\to\infty} E\left(\min\left\{\frac{p_0^*}{p_t},\ 1\right\}\right)^2.$$

Table 1
Estimates of $(1 + r_0)$, p_0^*, $\lambda_0, (1 + r_0)p_0^*$, and percentage of stockouts.[†]

	$1 + r$	p^*	λ	$(1 + r)p^*$	% stockouts
Cotton	1.0284	1.5307	0.9737	1.5741	38%
	(0.2041)	(0.3063)	(0.0006)	(0.0402)	
Maize	1.0647	1.4060	0.9836	1.4969	39%
	(0.1217)	(0.1711)	(0.0010)	(0.0626)	

[†] Estimated standard errors in parentheses.

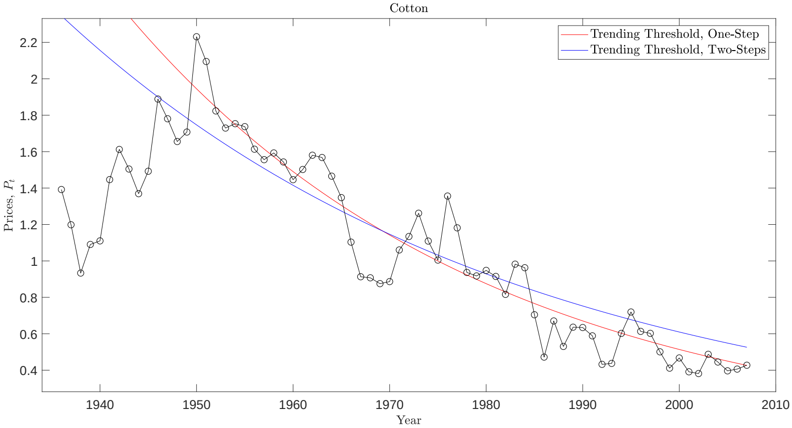
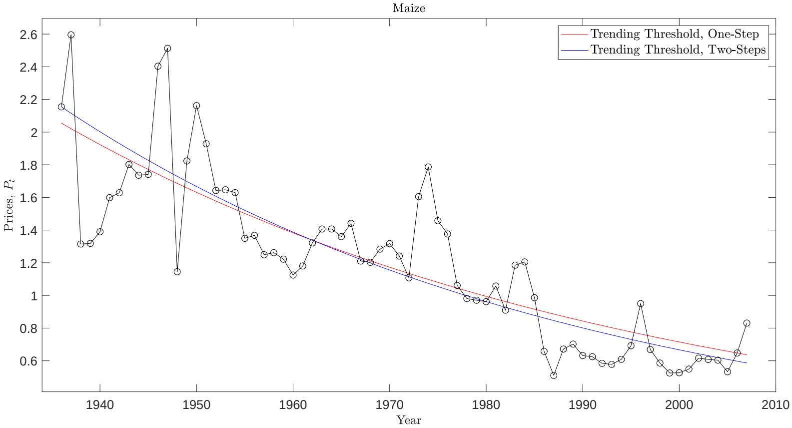


Figure: Annual prices and comparison of one-step and two-step estimation of the trending price threshold. 1936-2007



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The estimated trending price thresholds using preliminary detrending are annual price trends of -2.08 percent and -1.82 percent for cotton and maize respectively.

Using our new one-step approach the estimated annual price trends are -2.63 percent and -1.64 percent for cotton and maize respectively.

For the sample of cotton prices, the detrending method changes the sample splitting for 18 percent of the prices.

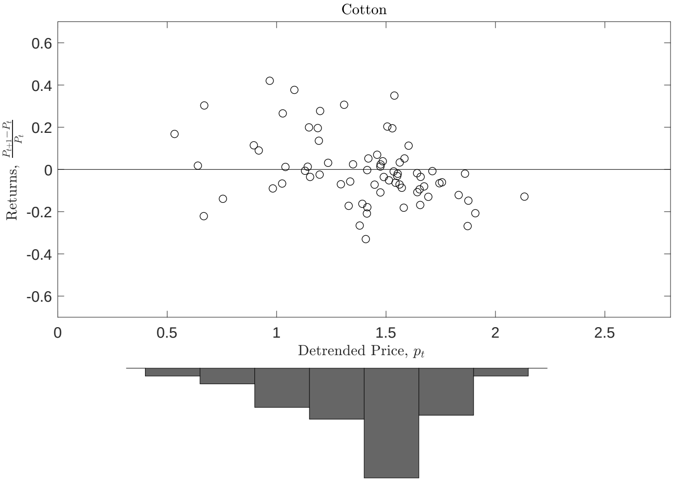
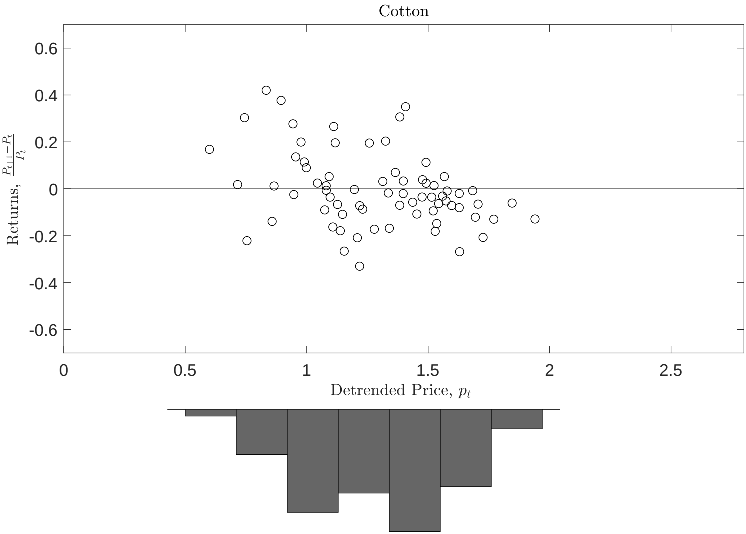


Figure: Cotton. One step Estimates. Annual proportional price changes between t and $t+1$ mapped from detrended price. 1936-2007



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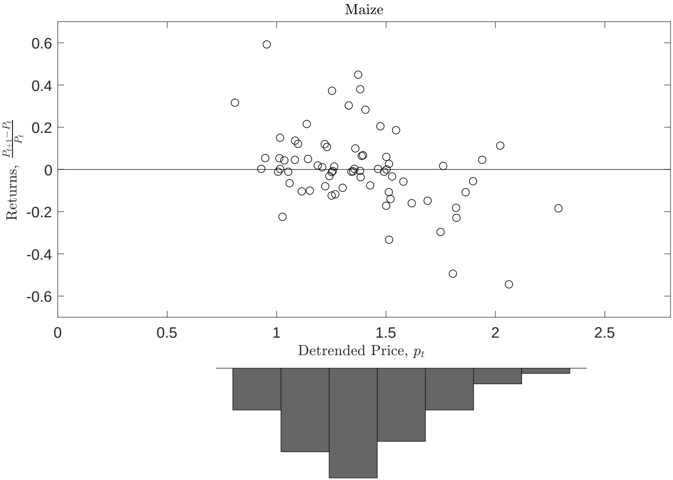

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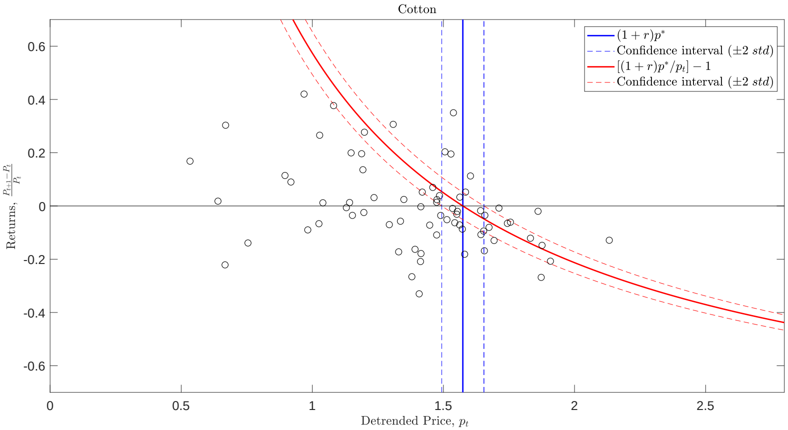
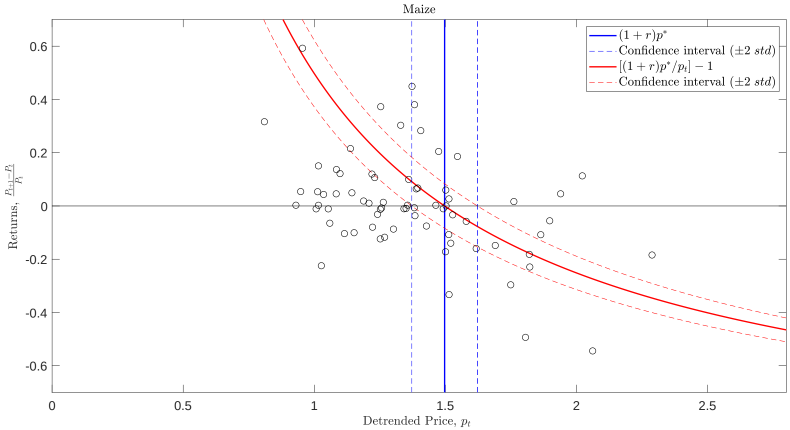


Figure: Cotton. Annual proportional price changes and model predictions



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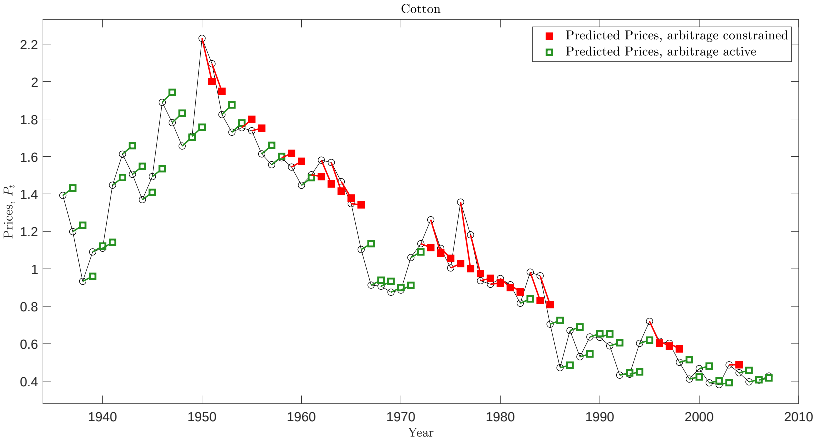

Figure: Annual prices and predicted prices. 1936-2007

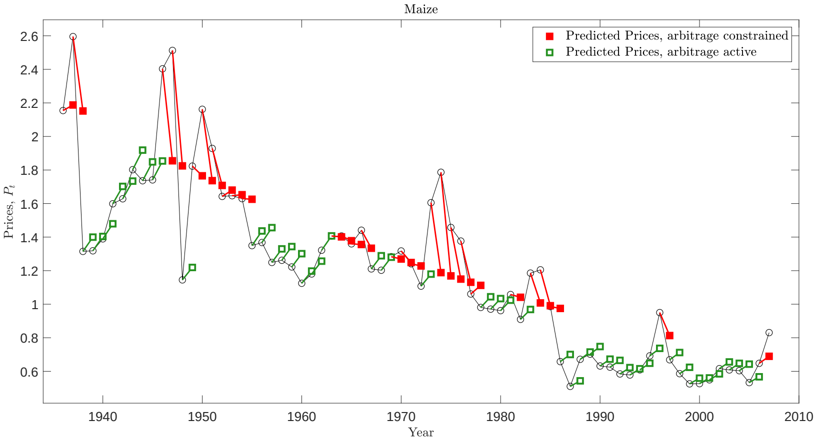

Figure: Annual prices and predicted prices. 1936-2007

Conclusions.

1. A seemingly negligible secular trend can create the illusion of unpredictability.

2. Illustrative samples of cotton and maize prices display alternation between two very different trend regimes.

2.1. In one regime, inventories are positive and price follows a *stochastic* trend with positive drift equal to the interest rate. The secular downward trend is latent in this regime.

2.2. In the other regime, the expected price change is a predictable jump to its conditional price expectation, which declines following the *secular* trend.

3. We implement a new one-step consistent estimator for the model, using nonstationary price data.

Conclusions (continued).

4. A model estimated on trending data can yield deterministic trends substantially different from trends estimated in a preliminary step, implying substantially different inferences about timings of price regimes
5. We estimate endogenous breaks between two regimes rather than inferring breaks as exogenous
6. We now we can get asymptotic standard errors for our estimates if the trending target, the "mean" to which prices in the stockout regime.

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